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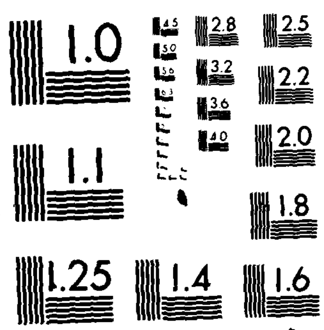

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Anti-Jam Analysis of Frequency Hopping M-ary Frequency Shift Keying Communication Systems in High Frequency Rayleigh Fading Channels

Report

to

NAVAL RESEARCH LABORATORY

(Contract Award No. N00014-80-K-0935)

for

HF COMMUNICATION NETWORK SIGNALS

USING CHANNEL EVALUATION DATA

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by

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PREFACE

This report to the Naval Research Laboratory is the Ph.D. dissertation of Dan Avidor. The work reported here was supported by the Naval Research Laboratory through contract ONR N00014-80-K-0935.

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ABSTRACT OF THE DISSERTATION

Anti-Jam Analysis of Frequency Hopping M-ary Frequency Shift Keying Communication Systems In High Frequency Rayleigh Fading Channels

by

Dan Avidor

Doctor of Philosophy in Engineering

University of California, Los Angeles, 1981

Professor Jim K. Omura, Chairman

A certain type of M-ary Frequency Shift Keying [MFSK] communication systems, which use Frequency Hopping [FH] to combat jamming is studied in this work. In particular, the performance of such systems over High Frequency [HF] sky-wave channels, when subjected to intentional jamming, is the main topic of this study. The channels considered are characterized by Rayleigh fading and additive Gaussian noise. To combat jamming the communication system hops over the total spread spectrum bandwidth, which is many times larger than the "instantaneous" bandwidth occupied by the MFSK signal. Located within the HF band and using sky-wave as the dominant propagation mode, the spread spectrum bandwidth is typically nonuniform, time dependent, and congested by many other users of the electromagnetic spectrum. In such an environment, Frequency Hopping [FH] MFSK systems seem to be more practical than "Direct Sequence" spectrum spreading techniques. A FH/MFSK system can

be easily programmed to use a noncontiguous band, thereby avoiding strong interfering signals (which are usually narrow-band signals) or friendly signals that should not be interfered with.

To make the best possible use of the available bandwidth, the HF/MFSK system requires channel related information and information related to the jammer. The receiver may probe the slowly varying parameters of the channel and jammer signal, and based upon this information, possibly supplemented by apriory knowledge of the jamming equipment, establish an "optimal hopping strategy". On a short term basis, the receiver may detect when a particular transmitted symbol is jammed or not, and use this information in the decoding process. In this work we study and compare the performance of anti-jam communication systems using this kind of information to that of similar systems that do not use such information.

Since coding and/or diversity is usually crucial in a fading environment, this study deals primarily with coded systems. A ganeral upper-bound on the coded-bit error probability is used as a basis for evaluating the performance of all anti-jam communication systems considered. This bound, which can be expressed as a function of the system's cutoff rate R_0, is independent of the specific coding employed, and therefore serves to decouple the coding aspects of the system from the remaining part of the communication system. It thus facilitates system analysis by allowing seperate treatment of each problem. This bound is used for optimizing various parameters of the systems under study. To compare alternative anti-jam systems, two common jamming techniques namely, noise jamming and multi-tone jamming, were selected

and the options open before the communication system and the jammer defined.

All the receivers that we consider in this study are conventional FH/MFSK noncoherent orthogonal receivers, a choice which seems justified by the robustness of noncoherent receivers compared to coherent systems. The receivers differ, though, by the type of processing conducted on the energy detector outputs, by the metrics used, and by the amount of auxiliary information related to jammer activity, which is available to them. In particular, we study the performance of Hard Decision receivers, Limiter and Quantizer-Limiter receivers. For each system we derive the corresponding upper bound as a function of ρ, where ρ is either the duty cycle of a pulsed jammer, or the fraction of the total spread spectrum bandwidth which is being jammed. We then find the value of ρ which maximizes the bound and finally compare the performance of the systems under study using the "worst case ρ" for each system. It is shown that the "worst case ρ" depends on the receiver being used by the "target communication system". For two specific cases we explicitly show its dependence on certain receiver parameters.

CHAPTER I

INTRODUCTION

1.1 The High Frequency Band

Throughout the most part of the wireless communication short history the High Frequency [HF] band namely, 3-30 MHz was the first line medium for long distance communication. Only recently have extensive development and construction of satellite Communication Systems [CS] overshadowed the crucial contribution of this medium to modern society. The HF band is no longer unique in some of its most important capabilities and yet it seems that it is not going to lose its dominant role in many civilian and military fields of activity. Moreover, research and technological achievements pertinent to HF communication technology promise enhanced use of this frequency band for reliable around the clock communication over large distances.

The HF band is relatively a complex communication medium. Some of the features characterising this medium are :

- Several propagation modes, notably ground-wave and sky-wave involving possibly several atmospheric reflecting layers giving rise to multipath propagation problems.
- The channel characteristics are usually time dependent.
- Generally, the channel has highly nonuniform spectral characteristics.
- The signals transmitted through a sky-wave path are subject to

fading and can in most cases be adequately described as having Rayleigh distributed envelope.

- The HF band is typically congested by high power transmitters and other types of man made emitters and is only loosly controlled by international regulatory authorities.

High frequency CS can easily be interfered with, since the interfering transmitter does not have to be in the vicinity of the receiving site, in order to cause effective interference. It is therefore generally accepted that Anti-Jam [AJ] capabilities are particularly important at the HF band. The existing interest in AJ capabilities of HF communication systems is further supported by the fact that long range HF links frequently serve as back-up systems to satellite relays, which are highly vulnerable to intelligent jamming.

1.2 Anti-Jam Capability

The basic key to combatting intentional jamming can be stated as follows :

> "Choose signal coordinates such that the jammer cannot achieve large jammer to signal power ratio in these coordinates".

If there are many signal coordinates available and only a small subset of them, not known to the jammer, is beeing used at any given time, then, the jammer is forced to jam all coordinates with little power in each or jam only few coordinates and leave the rest free. The signal coordinates to be used at any given instant are selected by a pseudo random [PN] sequence, known to the transmitter and intended receiver,

but not to the jammer.

Clearly, the more signal coordinates are available, the better the protection against jamming can be. Confined to a bandwidth W and duration T there are :

$$N = \begin{cases} 2WT \text{ coherent orthogonal waveforms} \\ WT \text{ noncoherent orthogonal waveforms} \end{cases}$$

For given W and T there are many possible ways to choose a set of coordinates, but most commonly it is done by one of the following two basic methods [10, 11] :

a. Direct Sequence spreading [DS]

b. Frequency Hopping [FH]

Hence the term "spread Spectrum" signals. Many hybrids of these two spreading techniques have also been devised, but their performance as an AJ protection tool, does not significantly differ from the basic ones.

The DS method is usually favored at high frequency bands (VHF and up) where wide bandwidths and line of sight propagation generally result in less spectrum crowding. At the HF band, however, it is difficult to maintain signal coherence over wide bandwidths, particularly when the dominant propagation mode is sky-wave. Several techniques have been proposed, which potentially can solve this problem, but some degradation in performance and considerable additional complexity is unavoidable. Moreover, a large number of conventional (narrow-band) signals may (and typically do) occupy the same band and interfere with the reception of the broad-band signal. In contrast, frequency Hopping system can use channels with relatively narrow coherence bandwidths.

They also have the following "practical" advantages:

- easier synchronization.
- wider spread bandwidths.
- do not require a contiguous band.
- are more compatible with other users of the same frequency band which usually transmit narrow band signals.

FH is therefore the most commonly used AJ techniques in the HF band.

1.3 AJ Design and Channel Probing

In a Rayleigh fading environment the error probability of an uncoded CS is roughly inversely proportional to the mean signal to noise ratio [SNR]. This is in sharp contrast to non-fading channels, in which the error probability decays exponentially when the SNR is increased. Consequently, even when no intentional jamming exists, it is extremely difficult and expensive to achieve a low error probability, say 10^{-6}, over a fading channel. When jamming exists the channel may be totally useless for the intended operation unless some form of coding is implemented (MFSK, $M > 2$ and diversity are in fact a simple form of coding). Therefore, this investigation is primarily concerned with overall performance of coded CSs under jamming.

It is widely recognized that modulation, receiver structure and coding techniques, that are well designed for an unjammed environment, do not necessarily perform well under jamming. Basic characteristics like signal wave-forms, demodulation techniques, coding, interleaving schemes etc., may be profoundly effected when requirements for anti-jam capability are introduced. Hence, the topic of AJ capability should

be considered in the design phase, rather than serve as one criterion of merit when comparing fixed parameter designs.

An efficient jamming operation usually requires measurements and analysis of various parameters of the target signal and the channel. Based on these measurements the optimal jamming strategy is determined. The jammer then monitors the target under jamming to assess its response if any, to his efforts. Likewise, it is intuitively clear that an improved AJ performance can be achieved by continuous probing / measuring the channel and jammer emission. This is the case in particular for HF sky-wave channels which are highly nonuniform, complex, time-varying and heavily congested by "innocent" users. There are many topics related to channel and jammer probing that should be considered. These include the following :

- What data should be collected?
- How should it be measured?
- How reliable are these measurements?
- How should the data be exploited?

In this study we analyse and compare the performance of several receiver structures which make use of "channel and jammer state knowledge" with that of receivers that do not use such information. For situations in which jammer's state information is not available, we introduce and study several different receiver structures intended to reduce the resulting degredation in performance. Throughout this study we assume that the receiver has "channel state information". For such cases we introduce optimal hopping strategy which takes advantage of the available data.

1.4 Outline of the Dissertation

In Chapter II we introduce all the main topics discussed in this dissertation. In particular, we introduce the channel model, define the "Slotted Channel", and discuss the concepts of CSI and JSI. The jamming modes studied in this work and the options open to the jammer are defined, as well as some basic assumptions related to the information that the jammer and the CS's operator have with respect to the capabilities of the other. These assumptions serve to define the "rules of the game". We then proceed to describe the various receiver structures to be studied and compared. The basic transmitter / receiver common to all systems studied in this work is presented. In section 5.2 we introduce additional definitions required in the sequel.

In chapter III we describe the basic analysis technique used throughout this work. We present the Chernoff bound for a general metric and show that when the ML metric is used, the Bhattacharyya bound results. The general bound parameter D and the cutoff rate R_0 are defined and R_0 is derived for the special case of M-ary symmetric channel.

The performance of six different MFSK receivers over a negligible background noise uniform channel is analyzed in chapter IV. For each receiver we derive the bound parameter D and the worst case duty cycle ρ. R_0 is computed under the worst jamming conditions for M = 2,4,8,16,32. These results are shown in figures 9a - 9e.

In chapter V we analyze the performance of four receivers operating over a nonuniform channel. For the Soft Decision receiver the

optimal metric weighing is derived and for all receivers the corresponding error bounds and the worst ρ is established.

In chapter VI we present several simple applications of the results derived in previous chapters. Using the union bound we derive simple bounds on the symbol and bit error ptobabilities P_s, and P_b of MFSK and m diversity MFSK. In order to show a typical application of the bound parameter D, we also introduce an m diversity orthogonal convolutional code and a numerical example. Several figures contained in this chapter compare the exact bit error probability to the corresponding bounds for several special cases.

The "Optimum Hopping Strategy" for the noise jamming case is derived in chapter VII. It is followed by a proof that the minimax solution, as derived for an uncoded system, is valid also for coded CS using Soft or Hard Decision receiver.

Chapter VIII deals with multi-tone jamming. It contains a general introduction to the subject, and a performance analysis of two receiver types under multi-tone jamming. Section 8.3 presents simple applications of results derived in chpater VIII. In section 8.4 we derive the "Optimal Hopping Streategy" for the multi-tone jamming case which yields results similar to those obtained in chapter VII for the noise jamming. Chapter IX contains some concluding remarks.

For easy reference two appendices were included. Appendix I contains the derivation of the symbol error probability of a noncoherent MFSK receiver in Rayleigh fading channel. Appendix II contains

the derivation of the symbol error probability of a noncoherent BFSK receiver in Rayleigh fading channel when hit by a multi-tone jam.

CHAPTER II

THE ANTI-JAM COMMUNICATION SYSTEM

2.1 Channel Characteristic and Probing

In this study we concentrate on the following problem: A certain segment of the High Frequency [HF] band (3-30 MHz) of bandwidth W supports an MFSK anti-jam [AJ] communication system. The signals transmitted are subject to Rayleigh fading and additive white Gaussian noise [AWGN]. The receiver is a noncoherent detection receiver, which uses Frequency Hopping [FH] to combat jamming.

Typically, a wide band HF channel (say, $W \geq 1$ MHz) is highly non-uniform. In this study we consider several channel models characterized by the following parameters:

- Average received signal power distribution across the bandwidth W.
- Jammer propagation loss distribution across the bandwidth W.
- Noise power and interfering signals distribution across the bandwidth W.

We divide the bandwidth W into L fixed sub-bands each of which supports one sub-channel. Each sub-band contains M tone positions and occupies a bandwidth of roughly M/T_c, where T_c is the "chip" duration. Hence :

$$L = \frac{W}{M/T_c} \tag{2.1}$$

where W is the total spread spectrum bandwidth. We assume that the fading is slow compared to T_c and uniform*. We also assume that each sub-channel fades independently and that each chip is independently hopped among many M tone sub-channels,and therefore, that any sequence of MFSK chips experiences independent fading and jamming noise in each chip.

As stated above, an improved AJ performance can be achieved by continuously probing/measuring the channel and the jammer emission. These measurements could be classified to long term and short term observations. Long term observations will be counted upon to supply the information related to background noise level and average signal power across the band, interfering signals, jammer power etc. We also consider two kinds of short term observations. The first depends on the ability of the receiver to detect the presence or absence of the jammer signal during each chip time interval, and modify the metric used by the decoder accordingly. The second kind is, in a sense, a second best alternative to the first and involves measurment of ρ, which is the duty cycle of a pulse jammer to be discussed below. Presumably, ρ can sometimes be measured even when the presence or absence of the jammer signal cannot be determined reliably enough for each chip signal individually. Receivers having channel parameter information will be refered to as having "Channel State Information" [CSI], whereas receivers

* By "uniform" we mean: Practically nonselective over a band which is at least as large as that occupied by a chip signal.

having CSI and also capable of detecting the presence of the jammer for each chip signal, will be referred to as having "Jammer State Information" [JSI].

For receivers / transmitters having CSI, we introduce the option of using non-uniform frequency hopping among M sub-bands. Coded CS are discussed, which use this information for establishing the optimal hopping strategy and also in the decoding process.

The simplest special case that we study is the uniform channel, for which the received average signal power, the noise power density and the jammer propagation loss are all uniform across the band W. Another, more general situation, is the "Slotted Channel" shown below:

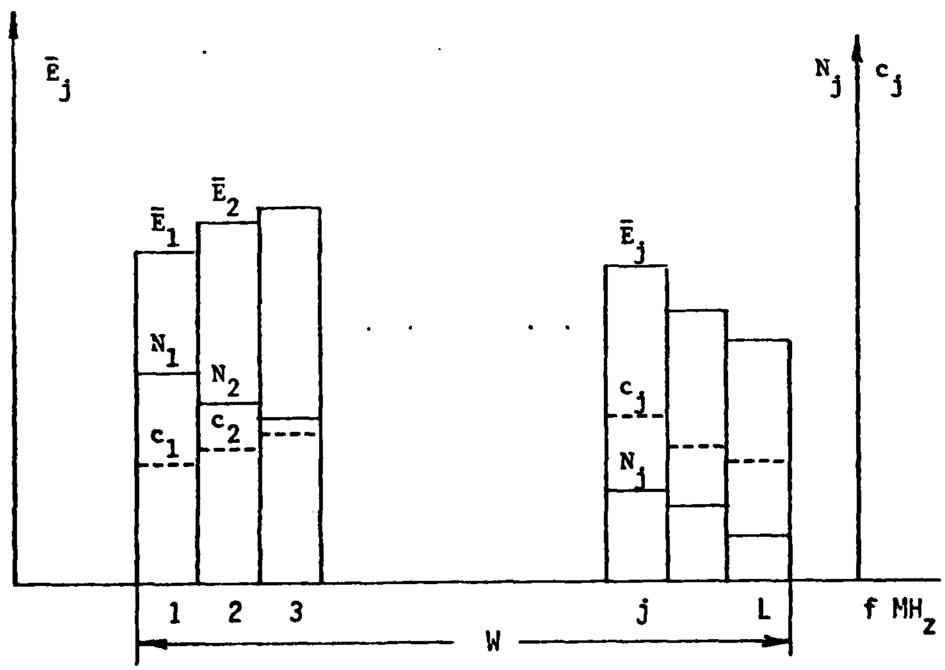


Figure 1 : The Slotted Channel

For each sub-channel we have a different noise level, average signal power and jammer propagation loss. Interfering signals, which "innocently occupy any of the sub-bands, are treated as an elevated level of background noise covering uniformly that specific sub-band. This can be justified by the fact that interfering tones experiencing Rayleigh fading appear as Gaussian noise at the receiver.

2.2 Jamming Techniques

Two basic jamming techniques are examined:

a. Noise jamming.

b. Multi-tone jamming.

We assume that the jammer has the option of distributing his total power J in any way across the bandwidth W. When considering a uniform channel, we assume partial-band jamming. In this case we denote by W_J the band "covered" by jamming noise, and define

$$\rho \triangleq \frac{W_J}{W} \tag{2.2}$$

where ρ is the jammed fraction of the total spread spectrum bandwidth W. Alternatively, we could regard ρ as being the duty cycle of a pulsed jammer. Performance-wise these two methods are equivalent due to the ideal interleaving that we assume. For nonuniform channels ρ can only be viewed as the duty cycle of a pulsed jammer.

For each receiver considered, we carry out the analysis for arbitrary ρ, and finally find the performance under the "worst case ρ".

2.3 Receivers Studied

Out of many possible receiver structures, that may seem appropriate, we have chosen to examine the following :

a. Hard decision with JSI.

b. Hard decision with no JSI.

c. Soft decision with no JSI.

d. Soft decision with JSI.

e. Quantizer-limiter receiver with no JSI.

f. Soft-decision-limiter receiver with no JSI.

The basic transmitter/receiver structure, common to all the systems considered in this study is the conventional noncoherent orthogonal FH/MFSK system. During each hop of duration T_c the modulator generates one out of M tones according to the $K = \log_2 M$ bits currently in the modulator register. The modulator output is shifted to the transmission frequency by the FH carrier generator, which is controlled by the PN sequence.

All the receivers that we consider contain the following :

1. A frequency dehopper.
2. A bank of M energy detectors.
3. M samplers.
4. A processing circuit or computing device to compute metrics.

The basic receiver is shown in figure 2 and a typical energy detector in figure 3 . Assuming that the i^{th} tone was sent during the n^{th} chip time interval: $(n-1)T_c < t < nT_c$, we have at the dehopper output :

$$H_i: \quad r(t) = A_n \cos(\omega_i t + \theta_n) + n(t) \quad , (n-1)T_c \leq t < nT_c$$

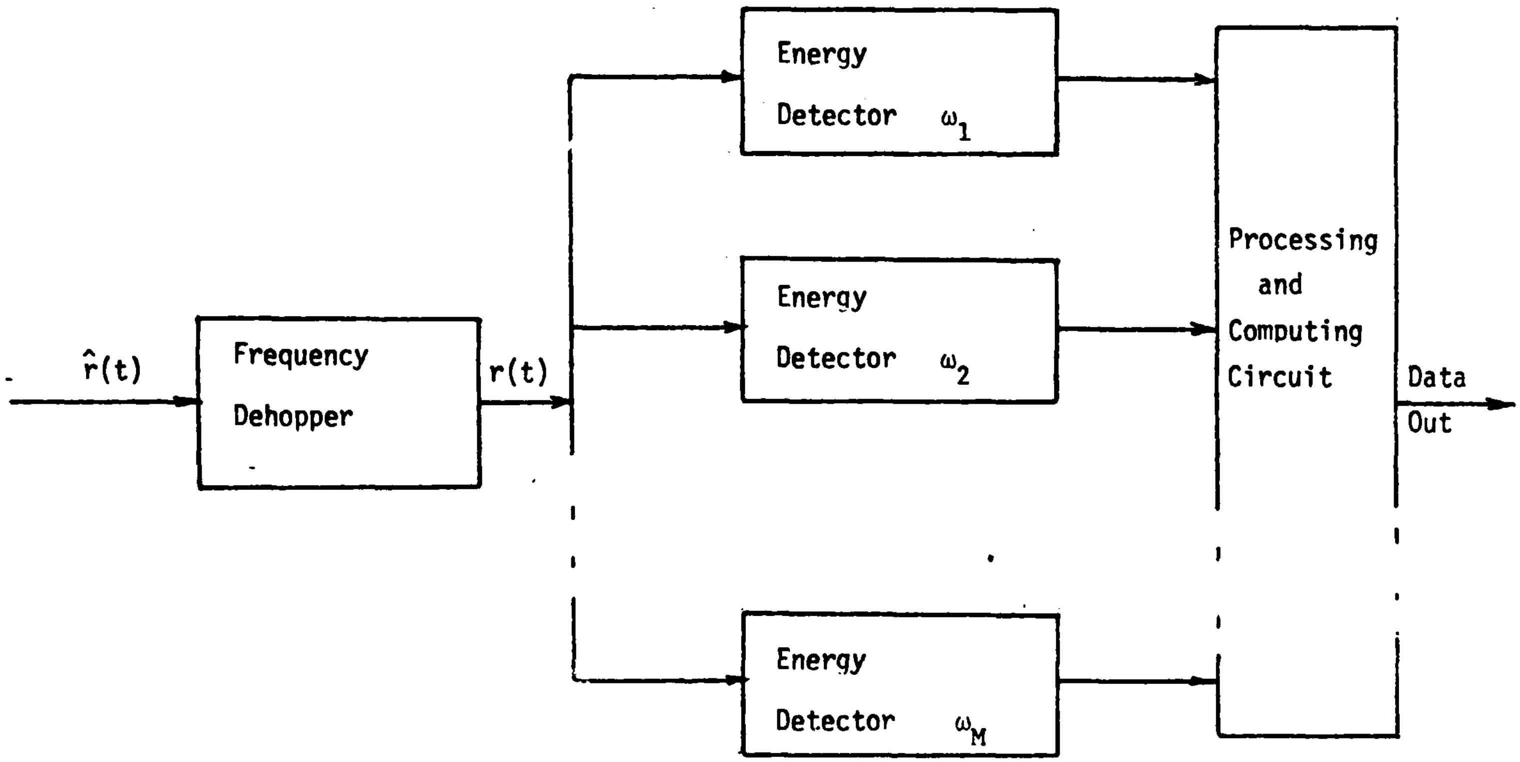
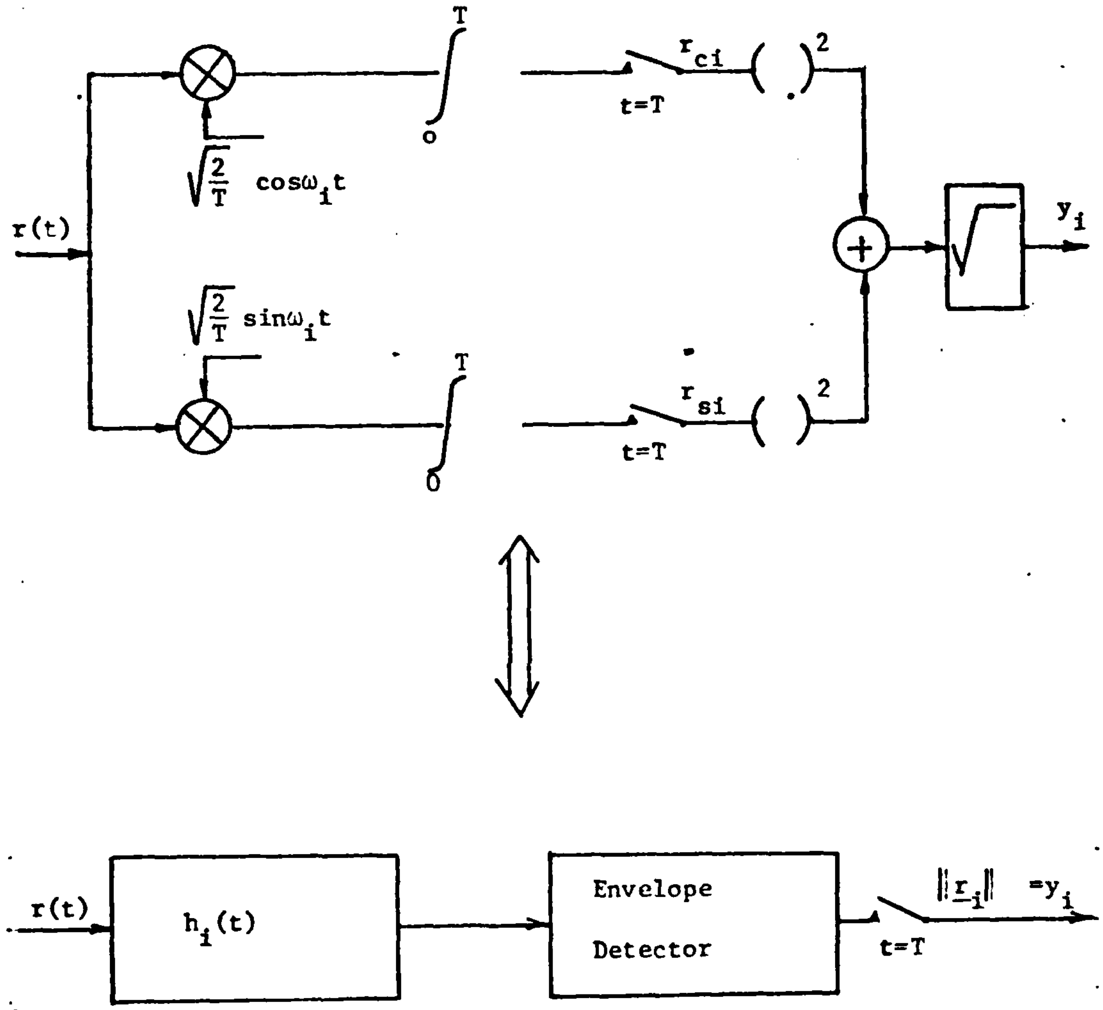


Figure 2 Th. Basic Receiver



$$h_i(t) = \sqrt{\frac{2}{T}}\ \cos\omega_i(T-t)$$

Figure 3 An Energy Detector

where T_c is the chip duration. Since a noncoherent Rayleigh fading channel is assumed, the probability density function of A_n and θ_n is:

$$P_{A_n}(a) = \frac{a}{\sigma_n^2} \exp\left\{-\frac{a^2}{2\sigma_n^2}\right\}$$

$$P_{\theta_n}(\theta) = \begin{cases} \frac{1}{2\pi} & , 0 \leqslant \theta < 2\pi \\ 0 & , \text{elsewhere} \end{cases}$$

where σ_n depends, in general, on the transmission frequency used during the n^{th} chip time.

2.4 Basic Rules

Our basic assumption is that the jammer knows everything there is to know about the channel, propagation loss, the equipment used by the "target CS", coding and interleaving techniques, with the exception only of the random PN sequence used to Hop/Dehop across the band. The CS's operator either knows all relevant parameters of the jammer, including the jammer's total power, or supplements his information with "pessimistic but reasonable" assumptions. This formulation establishes a lower bound on the performance of the CS under jamming. The problem is therefore a minimax problem: The CS operator chooses that set of parameters which achieves the best performance when the jammer excercises his jamming ability in the best possible way.

2.5 Definitions

We now introduce some additional definitions that will be required in the sequel.

When considering uniform and negligible noise channels, we denote the received jammer power by J, and define N_J as:

$$N_J = \frac{J}{W}$$

where W is the total spread spectrum bandwidth. When a uniform noisy channel is considered the single-sided spectral density of the additive white Gaussian noise is denoted N_0 and the total spectral density of the received noise is:

$$N_t = N_0 + N_J/\rho$$

in the jammed part of the band and:

$$N_t = N_0$$

in the unjammed part of the band.

Next consider conventional MFSK signaling over a Rayleigh fading channel with additive white Gaussian noise of single-sided spectral density N_0. Let H_j be the hypothesis that the j^{th} tone is sent and assume that the average signal energy at the receiver is $\bar{E}_c$. The square roots of the M energy detector outputs (see figures 2, 3) :

$$y_1, y_2, \ldots, y_M$$

are independent random variables with probability density functions:

$$P(y_j/H_j) = \frac{2y_j}{N_0+\bar{E}_c} \exp\left\{-\frac{y_j^2}{N_0+\bar{E}_c}\right\}$$

and

$$P(y_i/H_j) = \frac{2y_i}{N_o} \exp\left\{-\frac{y_i^2}{N_o}\right\} \qquad ; \quad i \neq j.$$

We now introduce a set of parameters indexed by the L sub-bands. The received dignal at the j^{th} sub-band is :

$$A_j \cos(\omega_{ji} t + \theta_{ji}) \qquad ; \quad j=1,\ldots,L \qquad ; \quad i=1,\ldots,M$$

where A_j is a Rayleigh distributed random variable:

$$P_{A_j}(a) = \frac{a}{\sigma_j^2} \exp\left\{-\frac{a^2}{2\sigma_j^2}\right\} \qquad ; \quad j=1,\ldots,L$$

which implies:

$$E\{A_j^2\} = 2\sigma_j^2$$

and

$$P_{\theta_{ji}}(\alpha) = \begin{cases} \frac{1}{2\pi} & ; \quad 0 \leq \alpha < 2\pi \\ 0 & ; \quad \text{elsewhere} \end{cases} \qquad ; \; j=1,\ldots,L \quad i=1,\ldots,M$$

Hence, the average received energy per chip when the j^{th} sub-band is used is;

$$\overline{E}_j = \sigma_j^2 T_c$$

when T_c is the chip duration. The noise distribution is given by:

$$\underline{N} = (N_1, N_2, \ldots, N_L)$$

where N_k is the single-sided spectral density for the k^{th} sub-band.

The jammer distributes total power $\underline{J}$ over the L sub-bands with distribution:

$$\underline{J} = (J_1, J_2, \ldots, J_L)$$

where

$$\sum_{j=1}^{L} J_j = J.$$

and J denotes in this case the total jammer transmission power.

The jammer's propagation loss, c_j, also depends on j. Hence, the contribution of the jammer to the noise power density of the j^{th} sub-band, denoted N_{Jj}, is

$$N_{Jj} = J_j \, c_j \, .$$

Whether pulsed or partial-band jammer is assumed, some received chips will be hit by the jammer and some will not.

The binary sequence:

$$\underline{Z} = (Z_1, \ldots, Z_m)$$

where

$$Z_i = \begin{cases} 0 \quad , & \text{the } i^{th} \text{ chip is not hit} \\ 1 \quad , & \text{the } i^{th} \text{ chip is hit by jammer} \end{cases}$$

specifies the jammed chips.

When hopping across the band, the hopping pattern will be defined by the vector $\underline{L}$:

$$\underline{L} = (j_1, j_2, \ldots, j_m)$$

where

$$j_k \in \{1, 2, \ldots, L\} \qquad ; \quad k = 1, \ldots, m$$

i.e., j_k specifies the sub-band used for the k^{th} chip. Note that for

any transmitted sequence $\underline{x}$, consisting of m MFSK chips, and denoted:

$$\underline{x} = (x^{(1)}, x^{(2)}, \ldots, x^{m})$$

there will be m sets of energy detector outputs:

$$\underline{y} = (\underline{y}^{(1)}, \underline{y}^{(2)}, \ldots, \underline{y}^{(m)})$$

where

$$\underline{y}^{(n)} = (y_1^{(n)}, y_2^{(n)}, \ldots y_M^{(n)}) \quad ; \quad n=1,\ldots,m$$

and $y_i^{(n)}$ is the i^{th} detector output at the end of the n^{th} chip time.

CHAPTER III

METHOD OF ANALYSIS AND PERFORMANCE EVALUATION

3. For uncoded CS we use the bit error probability as a performance criterion. For coded CS, however, exact bit error probability expre-ssions are typically difficult to obtain and upper bounds are used to evaluate performance.

3.1 A General Error Bound for the AJ Communication System

The coded AJ communication systems that we consider in this study are represented by the general model shown in figure 4 . We consider the sub-system shown inside the dotted lines as an equivalent memoryless channel available for sending coded data. The memoryless property is justified by the ideal interleaving that we assume. We then compute the catoff rate [3] R_0 of the equivalent channel, which represent the practically achievable reliable data rate per channel use. For any specific code we can then derive a bound on the coded bit error probability of the form :

$$P_b \leqslant B(R_0) \tag{3.1}$$

which is a function of the cutoff rate only. Since the function $B(R_0)$ is unique for each code, and R_0 is independent of the code used, we are able to decouple the coding from the rest of the CS. Thus, to evaluate various anti-jam CS we can simply compare the cutoff rates of these systems. By way of maximizing the cutoff rate, we also optimize certain parameters of the CS under study.

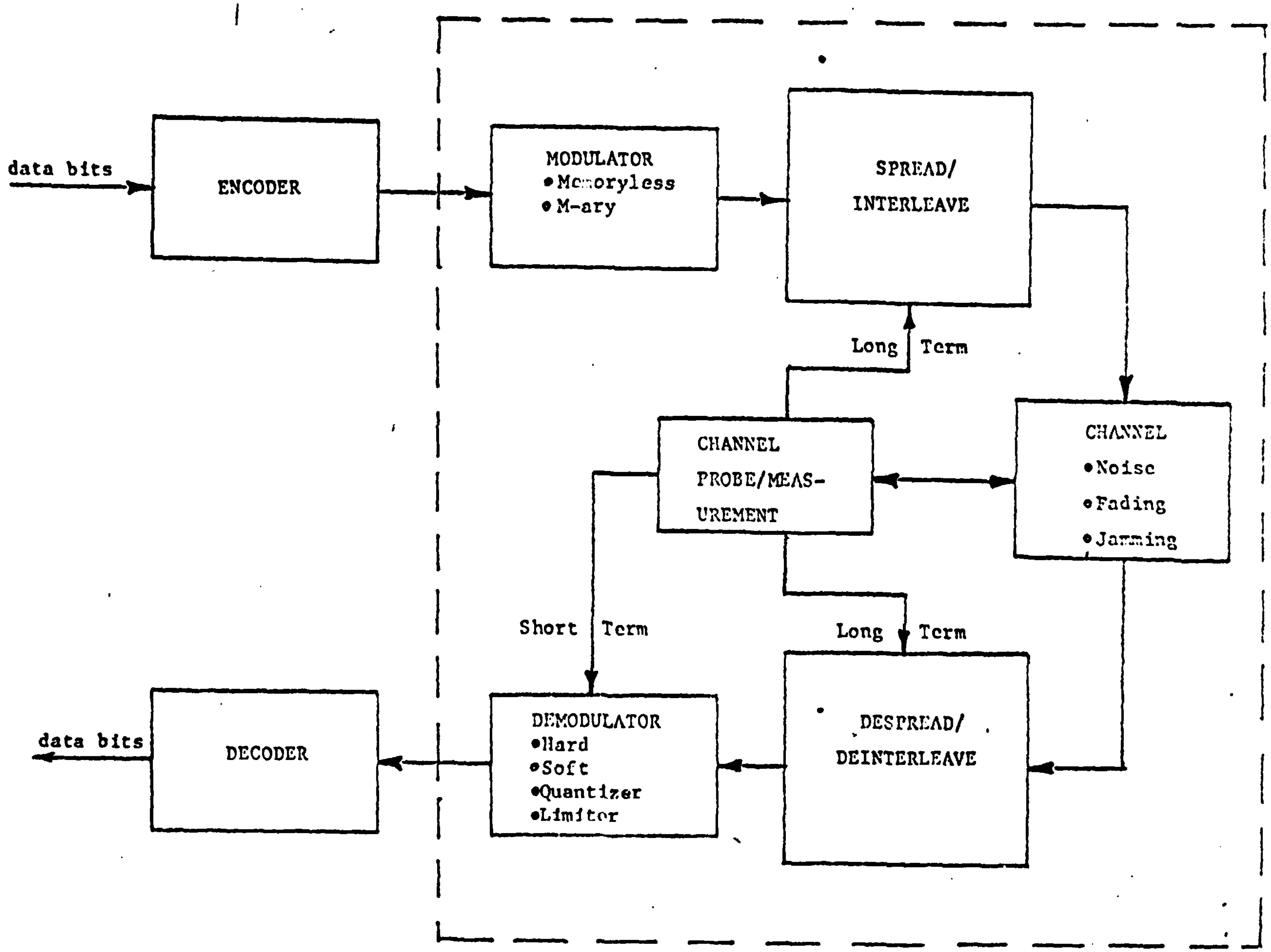


FIGURE 4 AJ SYSTEM

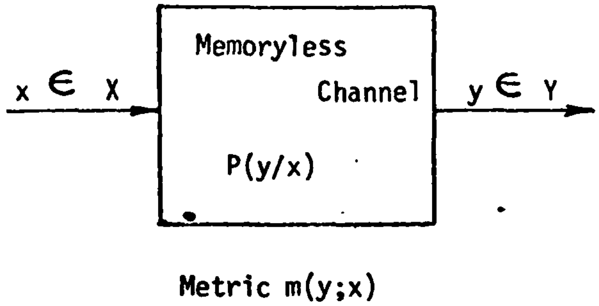


Figure 5 . Equivalent Memoryless Channel

The equivalent channel of figure 5 has input symbols which belong to the alphabet X and output symbols from the alphabet Y. The receiver uses the metric $m(y;x)$ to make decisions. Consider two sequences $\underline{x}$ & $\hat{\underline{x}} \in X^m$ and the pairwise error probability of the receiver choosing $\hat{\underline{x}}$ when $\underline{x}$ is transmitted, assuming that $\underline{x}$ & $\hat{\underline{x}}$ are the only possible transmitted sequences. We denote this probability by $P(\underline{x} \to \hat{\underline{x}})$.

Hence :

$$P(\underline{x} \to \hat{\underline{x}}) = P_r \left\{ \sum_{n=1}^{m} m(y_n;x_n) \leq \sum_{n=1}^{m} m(y_n;\hat{x}_n) \Big/ \underline{x} \right\}$$

$$= P_r \left\{ \sum_{n=1}^{m} [m(y_n;\hat{x}_n) - m(y_n;x_n)] \geq 0 \Big/ \underline{x} \right\}$$

We now use the Chernoff bound [3] with parameter $\lambda>0$:

$$P(\underline{x} \to \hat{\underline{x}}) \leq E\left[\exp\{ \lambda \sum_{n=1}^{m} [m(y_n;\hat{x}_n) - m(y_n;x_n)]\} / \underline{x} \right]$$

$$= \prod_{n=1}^{m} E\left[\exp\left\{ \lambda [m(y_n;\hat{x}_n) - m(y_n;x_n)] \right\} \Big/ x_n \right]$$

Defining :

$$D(x,\hat{x};\lambda) \triangleq E\left[\exp\{\lambda [m(y;\hat{x}) - m(y;x)]\} \Big/ x \right] \tag{3.2}$$

We obtain the Chernoff bound :

$$P(\underline{x} \to \hat{\underline{x}}) \leq \prod_{n=1}^{m} D(x_n,\hat{x}_n;\lambda) \tag{3.3}$$

Where :

$$D(x,x;\lambda) = 1 \quad \text{all} \quad x \in X \; . \tag{3.4}$$

3.2 Evaluation of R_o and the Bhattacharyya Bound

Suppose that all the components of the sequences $\underline{x}$ and $\underline{\hat{x}}$ are independently chosen according to some probability distribution $q(x)$, $x \in X$. The cutoff rate is then defined as:

$$R_o = \max_{o<\lambda} \max_{\underline{q}} R_o(\underline{q};\lambda) \tag{3.5}$$

where $R_o(\underline{q};\lambda)$ is given by the relation :

$$2^{-R_o(\underline{q};\lambda)} = E\left[D(x,\hat{x};\lambda)\right]$$

$$= \sum_{x} \sum_{\hat{x}} q(x)q(\hat{x})D(x,\hat{x};\lambda)$$

since, for MFSK the channel is symmetric and the input Alphabet size is $|X| = M$, we have

$$q(x) = \frac{1}{M} \quad , \quad x \in X$$

and

$$D(x,\hat{x};\lambda) = \begin{cases} D(\lambda) & , \hat{x} \neq x \\ 1 & , \hat{x} = x \end{cases} \tag{3.6}$$

Therefore:

$$2^{-R_o(\underline{q};\lambda)} = \frac{1+(M-1)D(\lambda)}{M}$$

and :

$$R_o = \log_2 M - \log_2[1+(M-1)D] \tag{3.7}$$

where :

$$D = \min_{\lambda>o} D(\lambda) \tag{3.8}$$

Equation 3.3 can then be written :

$$P(\underline{x} \rightarrow \hat{\underline{x}}) \leq D(\lambda)^{W(\underline{x},\hat{\underline{x}})}$$

When $W(\underline{x},\hat{\underline{x}})$ is the Hamming distance between $\underline{x}$ and $\hat{\underline{x}}$. To obtain the tightest bound we minimize this bound over $\lambda > 0$ to obtain :

$$P(\underline{x} \rightarrow \hat{\underline{x}}) \leq D^{W(\underline{x},\hat{\underline{x}})}$$

When using a specific code with many code-words it is then possible to evaluate the bit error probability of the code by union bounding all the pairwise error probabilities for each code-word and then averaging over all the code-words. This results in a bit error probability bound of the form :

$$P_b \leq G(D) \tag{3.9}$$

where $G(D)$ depends on the code being used. Since D can be expressed in terms of R_o, we also have the alternative form :

$$P_b \leq B(R_o)$$

Throughout this study we use the Chernoff bound to evaluate the performance of coded CS. For reasons which later become apparent, we consider receivers which employ several different metrics. If the metric happens to be the ML metric, which has the form :

$$m(y;x) = a\ell nP(y/x) + b \quad , \quad a > 0$$

we obtain :

$$D(x,\hat{x};\lambda) = E\left[\exp\{\lambda[m(y;\hat{x}) - m(y;x)]\} \;/x\right]$$

$$= \sum_y \exp\{\lambda a[\ell nP(y/\hat{x}) - \ell nP(y/x)]\}P(y/x)$$

$$= \sum_y P(y/x) \left[\frac{P(y/\hat{x})}{P(y/x)}\right]^{\lambda a}$$

The minimizing λ is usually $\lambda a = \frac{1}{2}$

Hence:

$$\min_{o<\lambda} D(x,\hat{x};\lambda) = \sum_{y} \sqrt{P(y/x)P(y/\hat{x})} \tag{3.10}$$

which is the Bhattacharyya bound [3].

CHAPTER IV

UNIFORM CHANNELS WITH NEGLIGIBLE BACKGROUND NOISE

4.1 Soft Decision Receiver With JSI

Our basic assumption is that the receiver is capable of detecting $\underline{Z}$ and is using it in the decoding process. In the uniform channel case we denote the total received jammer power by J and define:

$$N_J = J/W \tag{4.1}$$

The conditional density function of $\underline{W}$ given $\underline{x}$ and $\underline{Z}$ is then:

$$P_{mM}(\underline{y}/\underline{x},\underline{Z}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)},Z_n)$$

and

$$P_M(\underline{y}^{(n)}/x^{(n)},Z_n) = \prod_{k=1}^{M} P(y_k^{(n)}/x^{(n)},Z_n)$$

where

$$P(y_k^{(n)}/x^{(n)},Z_n=1) = \begin{cases} \dfrac{2y_k^{(n)}}{N_J/\rho+\bar{E}_c} \exp\left\{-\dfrac{{y_k^{(n)}}^2}{N_J/\rho+\bar{E}_c}\right\} & ; \quad x^{(n)}=k \\[2ex] \dfrac{2y_k^{(n)}}{N_J/\rho} \exp\left\{-\dfrac{{y_k^{(n)}}^2}{N_J/\rho}\right\} & ; \quad x^{(n)}\neq k \end{cases}$$

and

$$P(y_k^{(n)}/x^{(n)},Z_n=0) = \begin{cases} \dfrac{2y_k^{(n)}}{\bar{E}_c} \exp\left\{-\dfrac{{y_k^{(n)}}^2}{\bar{E}_c}\right\} & ; \quad x^{(n)}=k \\[2ex] \delta\left[y_k^{(n)}\right] & ; \quad x^{(n)}\neq k \end{cases}$$

Next define:

$$G(\underline{y},\underline{Z}) \triangleq \prod_{\substack{n=1 \\ n:Z_n=1}}^{m} \frac{N_J/\rho}{N_J/\rho+\bar{E}_c} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_J/\rho} \exp\left\{-\frac{{y_k^{(n)}}^2}{N_J/\rho}\right\}$$

$$\Delta(\underline{y};\underline{x},\underline{Z}) = \sum_{\substack{n=1\\ n:Z_n=1}}^{m} \frac{\bar{E}_c}{(N_J/\rho+\bar{E}_c)N_J/\rho} \, y_{x^{(n)}}^{(n)^2}$$

$$= \sum_{n=1}^{m} Z_n \, a \, y_{x^{(n)}}^{(n)^2}$$

where:

$$a \triangleq \frac{\bar{E}_c}{(N_J/\rho+\bar{E}_c)N_J/\rho}$$

Also let

$$F(\underline{y};\underline{x},\underline{Z}) = \prod_{\substack{n=1\\ n:Z_n=0}}^{m} \frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c} \exp\left\{-\frac{y_{x^{(n)}}^{(n)^2}}{\bar{E}_c}\right\} \prod_{\substack{k=1\\ k\neq x^{(n)}}}^{M} \delta(y_k^{(n)})$$

Then

$$P_{mM}(\underline{y}/\underline{x},\underline{Z}) = G(\underline{y},\underline{Z}) \exp\{\Delta(\underline{y};\underline{x},\underline{Z})\} F(\underline{y};\underline{x},\underline{Z})$$

The maximum likelihood [ML] receiver uses the total metric:

$$m(\underline{y};\underline{x}/\underline{Z}) = \ln P_{mM}(\underline{y}/\underline{x},\underline{Z})$$

$$= \ln G(\underline{y},\underline{Z}) + \Delta(\underline{y};\underline{x},\underline{Z}) + \ln F(\underline{y};\underline{x},\underline{Z})$$

But $G(\underline{y},\underline{Z})$ does not depend on $\underline{x}$. Furthermore, for all $\underline{x}$ such that

$\hat{x}^{(n)} \neq x^{(n)}$:

$$\ln\left[\frac{2y_{\hat{x}^{(n)}}^{(n)}}{\bar{E}_c} \exp\left\{-\frac{y_{\hat{x}^{(n)}}^{(n)^2}}{\bar{E}_c}\right\} \prod_{\substack{k=1\\ k\neq \hat{x}^{(n)}}}^{M} \delta(y_k^{(n)})\right] = -\infty$$

Hence, it suffices to compute:

$$\bar{m}(\underline{y};\underline{x}/\underline{Z}) = \sum_{n=1}^{m} Z_n y_{x^{(n)}}^{(n)^2} + \sum_{\substack{n=1\\ n:Z_n=0}}^{m} \ln\left(\prod_{\substack{k=1\\ k\neq x^{(n)}}}^{M} \delta(y_k^{(n)})\right) \tag{4.2}$$

for every sequence $\underline{x}$ C to determine the maximal likelihood sequence.

To find the performance of the receiver, we start with the Chernoff bound:

$$P(\underline{x} \to \hat{\underline{x}}) \leq E\left[\exp\left\{\lambda \sum_{n=1}^{m} [m(\underline{y}^{(n)};\hat{x}^{(n)}/Z_n) - m(\underline{y}^{(n)};x^{(n)}/Z_n)]\right\} / \underline{x}\right]$$

$$= \prod_{n=1}^{m} E\left[\exp\left\{\lambda [m(\underline{y}^{(n)};\hat{x}^{(n)}/Z_n) - m(\underline{y}^{(n)};x^{(n)}/Z_n)]\right\} / x^{(n)}\right]$$

$$= \prod_{n=1}^{m} D(\hat{x}^{(n)},x^{(n)};\lambda)$$

Where

$$D(\hat{x}^{(n)},x^{(n)};\lambda) \triangleq$$

$$= E\left[\exp\{\lambda [m(\underline{y}^{(n)};\hat{x}^{(n)}/Z_n) - m(\underline{y}^{(n)};x^{(n)}/Z_n)]\}/x^{(n)}\right]$$

Since the ML receiver uses the metric:

$$m(\underline{y}^{(n)};x^{(n)}/Z_n) = \ell n \; P_M(\underline{y}^{(n)}/x^{(n)},Z_n)$$

we obtain :

$$D(\hat{x}^{(n)},x^{(n)};\lambda) =$$

$$= E\left[\exp\{\lambda [\ell n \; P_M(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n) - \ell n \; P_M(\underline{y}^{(n)}/x^{(n)},Z_n)]\}/x^{(n)}\right]$$

$$= E\left\{\left[\frac{P_M(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n)}{P_M(\underline{y}^{(n)}/x^{(n)},Z_n)}\right]^{\lambda} / x^{(n)}\right\}$$

$$= E\left\{E\left\{\left[\frac{P_M(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n)}{P_M(\underline{y}^{(n)}/x^{(n)},Z_n)}\right]^{\lambda} / x^{(n)},Z_n\right\} / x^{(n)}\right\}$$

$$= E\left\{\int_0^{\infty} \cdots \int_0^{\infty} \frac{P_M(\underline{y}^{(n)}/\hat{x}^{(n)},Z_n)^{\lambda}}{P_M(\underline{y}^{(n)}/x^{(n)},Z_n)^{\lambda}} P_M(\underline{y}^{(n)}/x^{(n)},Z_n) dy_1^{(n)} \ldots dy_M^{(n)} / x^{(n)}\right\}$$

The value of λ which minimizes this bound is $\lambda = \frac{1}{2}$.

Hence:

$$\min_{o<\lambda} D(\hat{x}^{(n)}, x^{(n)}; \lambda) = D(\hat{x}^{(n)}, x^{(n)}; \tfrac{1}{2})$$

$$= E\left\{ \int_0^\infty \cdots \int_0^\infty \sqrt{P(\underline{y}^{(n)}/\hat{x}^{(n)}, Z_n) P(\underline{y}^{(n)}/x^{(n)}, Z_n)}\; dy_1^{(n)} \ldots dy_M^{(n)} \,/x^{(n)} \right\} \cdot \tag{4.3}$$

Which is the Bhattacharyya bound.

But since

$$P_{Z_n}(k) = \begin{cases} \rho & ; \quad k=1 \\ 1-\rho & ; \quad k=0 \end{cases}$$

this can be further reduced to:

$$D(\hat{x}^{(n)}, x^{(n)}; \tfrac{1}{2}) = \begin{cases} 1 & ; \quad x^{(n)} = \hat{x}^{(n)} \\ D & ; \quad x^{(n)} \neq \hat{x}^{(n)} \end{cases}$$

Where :

$$D = (1-\rho) \int_0^\infty \cdots \int_0^\infty \sqrt{P(\underline{y}^{(n)}/\hat{x}^{(n)}, 0) P(\underline{y}^{(n)}/x^{(n)}, 0)}\; dy_1^{(n)} \ldots dy_M^{(n)} +$$

$$+ \rho \int_0^\infty \cdots \int_0^\infty \sqrt{P(\underline{y}^{(n)}/\hat{x}^{(n)}, 1) P(\underline{y}^{(n)}/x^{(n)}, 1)}\; dy_1^{(n)} \ldots dy_M^{(n)}$$

Substituting the conditional probabilities that we have in this case, We obtain:

$$D = (1-\rho) \int_0^\infty \cdots \int_0^\infty \sqrt{\frac{4 y_{x^{(n)}}^{(n)} \, y_{\hat{x}^{(n)}}^{(n)}}{E_c^2} \exp\left\{ -\frac{y_{x^{(n)}}^{(n)2}}{E_c} - \frac{y_{\hat{x}^{(n)}}^{(n)2}}{E_c} \right\}} \quad x$$

$$\times \sqrt{\left[\prod_{\substack{k=1\\k\neq\hat{x}^{(n)}}}^{M} \delta(y_k^{(n)})\right]\left[\prod_{\substack{k=1\\k\neq x^{(n)}}}^{M} \delta(y_k^{(n)})\right]}\; dy_1^{(n)}\ldots dy_M^{(n)} +$$

$$+ \rho \int_0^\infty \!\!\ldots\!\! \int_0^\infty \sqrt{\frac{4y_{x^{(n)}}^{(n)}}{(\bar{E}_c+N_J/\rho)^2} \exp\left\{-\frac{y_{x^{(n)}}^{(n)^2}+y_{\hat{x}^{(n)}}^{(n)^2}}{E_c+N_J/\rho}\right\}\left[\prod_{\substack{k=1\\k\neq x^{(n)}}}^{M} \frac{2y_k^{(n)}}{N_J/\rho} \exp\left\{-\frac{y_k^{(n)^2}}{N_J/\rho}\right\}\right]} \times$$

$$\times \sqrt{\left[\prod_{\substack{j=1\\j\neq\hat{x}^{(n)}}}^{M} \frac{2y_j^{(n)}}{N_J/\rho} \exp\left\{-\frac{y_j^{(n)^2}}{N_J/\rho}\right\}\right]}\; dy_1^{(n)}\ldots dy_M^{(n)}$$

The first integral is clearly zero. The second can be integrated over $y_i^{(n)}$,

$$i=1,\ldots,M \quad ; \quad i\neq x^{(n)} \quad ; \quad i\neq\hat{x}^{(n)}$$

and finally reduces to:

$$\rho \left\{\int_0^\infty \frac{2y}{\sqrt{N_J/\rho(\bar{E}_c+N_J/\rho)}} \exp\left\{-\frac{y^2}{2}\left(\frac{1}{N_J/\rho+\bar{E}_c}+\frac{1}{N_J/\rho}\right)\right\} dy\right\}^2$$

$$= \rho\, \frac{4N_J/\rho(N_J/\rho+\bar{E}_c)}{(2N_J/\rho+\bar{E}_c)^2} \quad = \quad \rho\, \frac{4(1+\rho\bar{E}_c/N_J)}{(2+\rho\bar{E}_c/N_J)^2}$$

Therefore

$$P(\underline{x} \rightarrow \hat{\underline{x}}) \leq D^{W(\underline{x};\hat{\underline{x}})} =$$

$$= \frac{4\rho(1+\rho E_c/N_J)}{(2+\rho E_c/N_J)^2}^{W(\underline{x};\underline{x})} \qquad (4.4)$$

It is easily verified that $\rho = 1$ maximizes this bound, i.e., continuous jamming (broadband-jamming) is the worst case jamming for this receiver.

Or

$$D_{wc} = \max_{0<\rho\leq 1} D = \frac{4(1+E_c/N_J)}{(2+E_c/N_J)^2} \qquad (4.5)$$

4.2 Soft Decision Receiver with No JSI

Since we have just seen that continuous jamming is the "worst case jamming" for the receiver analyzed in 4.1, and since the receiver we presently consider does not have JSI, we are tempted to try the simple metric:

$$m(\underline{y};\underline{x}) = \sum_{n=1}^{n} y_{x^{(n)}}^{(n)^2} \qquad (4.6)$$

which is just an equal weight summation of the energy detector outputs, which correspond to a sequence $\underline{x}$.

Hence

$$P(\underline{x} \to \hat{\underline{x}}) \leq E\left[\exp\left\{\lambda \sum_{n=1}^{m}\left(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2}\right)\right\} \Big/ \underline{x}\right]$$

$$= \prod_{n=1}^{m} E\left[\exp\left\{\lambda\left(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2}\right)\right\} \Big/ x^{(n)}\right]$$

$$\therefore \quad D(x^{(n)},\hat{x}^{(n)};\lambda) = E\left[\exp\left\{\lambda\left(y_{x^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2}\right)\right\} \Big/ x^{(n)}\right]$$

$$= (1-\rho)E\left[\exp\left\{\lambda\left(y_{\hat{x}(n)}^{(n)^2} - y_{x(n)}^{(n)^2}\right)\right\} / x^{(n)}, Z_n=0\right] +$$

$$+ \rho E\left[\exp\left\{\lambda\left(y_{\hat{x}(n)}^{(n)^2} - y_{x(n)}^{(n)^2}\right)\right\} / x^{(n)}, Z_n=1\right] .$$

But given $x^{(n)}$, $\hat{x}^{(n)} \neq x^{(n)}$ and Z_n , $y_{\hat{x}(n)}^{(n)}$ & $y_{x(n)}^{(n)}$ are statistically independent.

Hence, for $\hat{x}^{(n)} \neq x^{(n)}$,

$$D(\hat{x}^{(n)}, x^{(n)}; \lambda) =$$

$$= (1-\rho) \quad E\left[\exp\left\{\lambda y_{\hat{x}(n)}^{(n)^2}\right\} / x^{(n)}, Z_n=0\right] \cdot E\left[\exp\left\{-\lambda y_{x(n)}^{(n)^2}\right\} / x^{(n)}, Z_n=0\right]$$

$$+ \rho E\left[\exp\left\{\lambda y_{x(n)}^{(n)^2}\right\} / x^{(n)}, Z_n=1\right] \cdot E\left[\exp\left\{-\lambda y_{x(n)}^{(n)^2}\right\} / x^{(n)}, Z_n=1\right]$$

We now use the fact that for a Gaussian random variable $x \sim N(0,\sigma_x^2)$:

$$E[\exp\{\lambda x^2\}] = \frac{1}{\sqrt{1-2\lambda\sigma_x^2}} \quad ; \quad \lambda < \frac{1}{2\sigma_x^2} \qquad (4.7)$$

$$E[\exp\{-\lambda x^2\}] = \frac{1}{\sqrt{1+2\lambda\sigma_x^2}} \quad ; \quad \lambda > -\frac{1}{2\sigma_x^2} \qquad (4.8)$$

Since

$$y_k^{(n)^2} = r_{ck}^{(n)^2} + r_{sk}^{(n)^2}$$

and since, given $x^{(n)}$ and Z_n, $r_{ck}^{(n)}$ & $r_{sk}^{(n)}$ are Gaussian and statistically independent (see figure 3), we obtain:

$$E\left[\exp\left\{\lambda y_k^{(n)^2}\right\}\right] = E\left[\exp\left\{\lambda\left(r_{ck}^{(n)^2} + r_{sk}^{(n)^2}\right)\right\}\right]$$

$$= E\left[\exp\left\{\lambda r_{ck}^{(n)^2}\right\}\right] \cdot E\left[\exp\left\{\lambda r_{sk}^{(n)^2}\right\}\right]$$

$$\therefore \quad D(x^{(n)},\hat{x}^{(n)};\lambda) =$$

$$= \frac{1-\rho}{(1+\lambda E_c)} + \frac{\rho}{(1-\lambda N_J/\rho)[1+\lambda(N_J/\rho+E_c)]} = D(\lambda) \tag{4.9}$$

$$0 < \lambda < \frac{\rho}{N_J}$$

Now we want to find the worst case D by taking the maximum over ρ, $0 < \rho \leqslant 1$ and the minimum over λ: $0 < \lambda < \frac{\rho}{N_J}$ i.e.,

$$D_{Wc} = \max_{0<\rho\leqslant 1} \quad \min_{0<\lambda<\rho/N_J} \quad D(\lambda)$$

Note, however, that the condition:

$$\lambda < \frac{\rho}{N_J}$$

implies that for any allowable value of λ, the first term, namely:

$$\frac{1-\rho}{1+\lambda E_c}$$

approaches 1 as $\rho \to 0$, and therefore,

$$\min_{0<\lambda<\rho/N_J} D(\lambda) \xrightarrow{\rho \to 0} 1$$

We expect that in general, the Soft Decision receiver using this metric, has poor performance under a low duty cycle jammer.

Although the receiver now considered does not have JSI, it may still know ρ. In such a case the receiver can use the ML metric. We want now to find this metric and to analyze the performance of a Soft Decision receiver using the ML metric. The conditional probability of $\underline{y}$ given the transmitted sequence $\underline{x}$ is:

$$P_{mM}(\underline{y}/\underline{x}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)}) =$$

$$= \prod_{n=1}^{m}\left[\sum_{k=0}^{1} P_M(\underline{y}^{(n)}/x^{(n)},k)P_{Z_n}(k)\right] \tag{4.10}$$

$$= \prod_{n=1}^{m}\left[P_M(\underline{y}^{(n)}/x^{(n)},0)\ (1-\rho) + P_M(\underline{y}^{(n)}/x^{(n)},1)\rho\right] \tag{4.11}$$

But

$$P_M(\underline{y}^{(n)}/x^{(n)},0) = \frac{2y^{(n)}_{x^{(n)}}}{E_c} \exp\left\{-\frac{{y^{(n)}_{x^{(n)}}}^2}{E_c}\right\} \prod_{\substack{k=1\\k\neq x^{(n)}}}^{M} \delta(y_k^{(n)})$$

and

$$P_M(\underline{y}^{(n)}/x^{(n)},1) =$$

$$= \frac{2y^{(n)}_{x^{(n)}}}{E_c+N_J/\rho} \exp\left\{-\frac{{y^{(n)}_{x^{(n)}}}^2}{N_J/\rho+E_c}\right\} \prod_{\substack{k=1\\k\neq x^{(n)}}}^{M} \frac{2y_k^{(n)}}{N_J/\rho} \exp\left\{-\frac{{y_k^{(n)}}^2}{N_J/\rho}\right\}$$

Hence:

$$P_{mM}(\underline{y}/\underline{x}) =$$

$$= \prod_{n=1}^{m} \left[(1-\rho) \frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c} \exp\left\{ - \frac{y_{x^{(n)}}^{(n)^2}}{\bar{E}_c} \right\} \prod_{\substack{k=1 \\ k \neq x^{(n)}}}^{M} \delta(y_k^{(n)}) \right.$$

$$\left. + \rho \frac{N_J/\rho}{\bar{E}_c + N_J/\rho} \exp\left\{ \frac{y_{x^{(n)}}^{(n)^2} \bar{E}_c}{(\bar{E}_c + N_J/\rho) N_J/\rho} \right\} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_J/\rho} \exp\left\{ - \frac{y_k^{(n)^2}}{N_J/\rho} \right\} \right]$$

Let

$$G(\underline{y}^{(n)}) \triangleq \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_J/\rho} \exp\left\{ - \frac{y_k^{(n)^2}}{N_J/\rho} \right\}$$

Then

$$P_{mM}(\underline{y}/\underline{x}) = \prod_{n=1}^{m} \left[(1-\rho) \frac{2y_{x^{(n)}}^{(n)}}{\bar{E}_c} \exp\left\{ - \frac{y_{x^{(n)}}^{(n)^2}}{\bar{E}_c} \right\} \prod_{\substack{k=1 \\ k \neq x^{(n)}}}^{M} \delta(y_k^{(n)}) + \right.$$

$$\left. + \rho \frac{N_J/\rho}{N_J/\rho + \bar{E}_c} G(\underline{y}^{(n)}) \exp\left\{ \frac{y_{x^{(n)}}^{(n)^2} \bar{E}_c}{(\bar{E}_c + N_J/\rho) N_J/\rho} \right\} \right]$$

Now we take:

$$m(\underline{y};\underline{x}) = \ln P_{mM}(\underline{y}/\underline{x}) =$$

$$= \sum_{n=1}^{m} \ln \left[(1-\rho) \frac{2y_{x(n)}^{(n)}}{\bar{E}_c} \exp\left\{ - \frac{y_{x^{(n)}}^{(n)^2}}{\bar{E}_c} \right\} \prod_{\substack{k=1 \\ k \neq x^{(n)}}}^{M} \delta(y_k^{(n)}) + \right.$$

$$\left. + \rho \frac{N_J/\rho}{\bar{E}_c + N_J/\rho} G(\underline{y}^{(n)}) \exp\left\{ \frac{y_{x^{(n)}}^{(n)^2} \bar{E}_c}{(\bar{E}_c + N_J/\rho) N_J/\rho} \right\} \right] \tag{4.12}$$

i.e., the receiver uses the ML metric. Therefore, as we have seen above, the Chernoff bound reduces to the Bhattacharyya bound:

$$P_r(\underline{x} \to \underline{\hat{x}}) \leqslant D^{W(\underline{x};\underline{\hat{x}})} \quad ; \quad \text{where}$$

$$D = \int_0^\infty \cdots \int_0^\infty \sqrt{P_M(\underline{y}^{(n)}/x^{(n)})P_M(\underline{y}^{(n)}/x^{(n)})} \quad dy_1^{(n)} \ldots dy_M^{(n)}$$

Substituting $P_M(\underline{y}^{(n)}/x^{(n)})$ & $P_M(\underline{y}^{(n)}/\hat{x}^{(n)})$ we obtain:

$$= \int_0^\infty .. \int_0^\infty \rho \frac{N_J/\rho}{N_J/\rho+\bar{E}_c} G(\underline{y}^{(n)}) \exp\left\{\frac{\left|y_{x^{(n)}}^{(n)2}+y_{\hat{x}^{(n)}}^{(n)2}\right|\bar{E}_c}{2N_J/\rho(\bar{E}_c+N_J/\rho)}\right\} dy_1^{(n)} \ldots dy_M^{(n)}$$

Carrying out the integration over $y_i^{(n)}$, $i=1,\ldots,M$, $i\neq x^{(n)}$, $i\neq\hat{x}^{(n)}$

We obtain the double integral:

$$D = \rho\left[\int_0^\infty \frac{2y}{\sqrt{(N_J/\rho+\bar{E}_c)N_J/\rho}} \exp\left\{y^2\left(-\frac{1}{N_J/\rho}+\frac{\bar{E}_c}{(N_J/\rho+\bar{E}_c)2N_J/\rho}\right)\right\}dy\right]^2$$

$$= \rho\left[\int_0^\infty \frac{2y}{\sqrt{(N_J/\rho+\bar{E}_c)N_J/\rho}} \exp\left\{-y^2\frac{\bar{E}_c+2N_J/\rho}{(N_J/\rho+\bar{E}_c)2N_J/\rho}\right\} dy\right]^2$$

$$= \rho\frac{4\left(1+\frac{\bar{E}_c}{N_J/\rho}\right)}{\left(2+\frac{\bar{E}_c}{N_J/\rho}\right)^2} \qquad (4.13)$$

This is exactly the value of D obtained above for the Soft Decision receiver having JSI. We conclude that a Soft Decision receiver with no

JSI, but knowing ρ, performs exactly as well as a Soft Decision receiver having JSI, provided the background noise is negligible.

As before, the worst case ρ is $\rho=1$, in which case:

$$\max_{0<\rho\leq 1} D = \frac{4\left(1+\frac{E_c}{N_J}\right)}{\left(2+\frac{E_c}{N_J}\right)^2} \tag{4.14}$$

4.3 Hard Decision receiver with JSI

The input and output alphabets of the channel are:

$$X = Y \in \{1,2,\ldots,M\}$$

and the conditional probability

$$P(y/x,Z) = \begin{cases} 1 & ; \quad y=x \quad , \quad Z=0 \\ 0 & ; \quad y\neq x \quad , \quad Z=0 \\ 1-\varepsilon & ; \quad y=x \quad , \quad Z=1 \\ \frac{\varepsilon}{M-1} & ; \quad y\neq x \quad , \quad Z=1 \end{cases} \tag{4.15}$$

Where :

$$\varepsilon(\rho) = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+\frac{\rho E_c}{N_J}\right)}$$

Since the channel is memoryless

$$P_m(\underline{y}/\underline{x},\underline{Z}) = \prod_{n=1}^{m} P(y_n/x_n,Z_n)$$

We choose the metric

$$m(y_n;x_n/Z_n) = \ln P(y_n/x_n,Z_n)$$

which is the ML metric.

Hence, the Chernoff bound reduces to the Bhattacharyya bound:

$$P(\underline{x} \to \underline{\hat{x}}) \leq D^{W(\underline{x};\underline{\hat{x}})}$$

Where

$$D \triangleq E\left\{ \sum_{y} \sqrt{P(y_n/\hat{x}_n,Z_n)P(y_n/x_n,Z_n)} \;/x_n \right\}$$

$$= (1-\rho) \sum_{y} \sqrt{P(y_n/x_n,0)P(y_n/\hat{x}_n,0)} + \rho \sum_{\substack{y \\ x_n \neq \hat{x}_n}} \sqrt{P(y_n/x_n,1)P(y_n/\hat{x}_n,1)}$$

But, for $x_n \neq x_n$,

$$P(y_n/x_n,0) \neq 0 \Longrightarrow P(y_n/\hat{x}_n,0) = 0$$

Therefore:

$$D = \rho \sum_{y_n} \sqrt{P(y_n/x_n,1)P(y_n/\hat{x}_n,1)}$$

$$= \rho\left[2\sqrt{\frac{[1-\varepsilon(\rho)]\varepsilon(\rho)}{M-1}} + \frac{M-2}{M-1}\varepsilon(\rho)\right] \qquad (4.16)$$

It is easy to verify that $\rho=1$ maximizes this bound, i.e., continuous jamming (broadband jamming) is the worst case jamming for this receiver also. Substituting $\rho=1$ in the bound 4.16, we obtain:

$$D_{WC} = 2\sqrt{\frac{\varepsilon(1-\varepsilon)}{M-1}} + \frac{M-2}{M-1}\varepsilon \qquad (4.17)$$

where ε denotes $\varepsilon(1)$.

4.4 Hard Decision Receiver With No JSI

The input and output alphabets are:

$$X = Y \in \{1,2,\ldots,M\}$$

but the conditional probability function is now:

$$P(y/x) = \sum_{z=0}^{1} P(y/x,z)P_Z(z) =$$

$$= \quad (1-\rho)P(y/x,0) + \rho P(y/x,1)$$

But:

$$P(y/x,0) = \begin{cases} 1 & ; \quad x = y \\ 0 & ; \quad x \neq y \end{cases}$$

$$P(y/x,1) = \begin{cases} 1 - \varepsilon(\rho) & ; \quad x = y \\ \frac{\varepsilon(\rho)}{M-1} & ; \quad x \neq y \end{cases}$$

Where

$$\varepsilon(\rho) = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k(1 + \rho\bar{E}_c/N_J)}$$

Hence:

$$P(y/x) = \begin{cases} 1 - \rho\varepsilon(\rho) & ; \quad y = x \\ \frac{\rho\varepsilon(\rho)}{M-1} & ; \quad y \neq x \end{cases} \tag{4.18}$$

In this case we use the simple metric

$$m(y_n;x_n) = - W(y_n,x_n)$$

This is, in fact, the ML metric, since it can be written in the form

$$m(y_n;x_n) = a\ell n P(y_n/x_n) + b; \qquad a > 0$$

Hence, we can use the Bhattacharyya bound:

$$P\,(\underline{x} \to \hat{\underline{x}}) \leq D^{W(\underline{x};\hat{\underline{x}})}$$

where

$$D = \sum_{y} \sqrt{P(y/x_n)P(y/\hat{x}_n)} \;=\; ; \quad \hat{x}_n \neq x_n$$

$$= 2\sqrt{\frac{\rho\varepsilon(\rho)[1-\rho\varepsilon(\rho)]}{M-1}} + \rho\,\frac{M-2}{M-1}\,\varepsilon(\rho) \tag{4.19}$$

It is easily verified that $\rho = 1$ maximizes D for this receiver also i.e.,

$$D_{Wc} = \max_{0<\rho\leq 1} D = 2\sqrt{\frac{\varepsilon(1-\varepsilon)}{M-1}} + \frac{M-2}{M-1}\,\varepsilon \tag{4.20}$$

where

$$\varepsilon \triangleq \varepsilon(1).$$

Clearly, the results of the receivers discussed in section 4.3 and in this section should coincide for $\rho=1$, and indeed we see that the corresponding bounds are also the same.

4.5 The Quantizer-Limiter Receiver with No JSI

This section contains two parts. In sub-section 4.5.1 we analyze the performance of the Quantizer-Limiter receiver under broadband jamming ($\rho=1$), and find the optimal quantization-step size T, which is a function of both E_c and N_J. In this derivation N_J could equally well represent the spectral density of the noise introduced by a noisy channel, or the combined density of channel noise and jammer generated noise. In section 4.5.2 we analyze the performance of the Quantizer-Limiter receiver under partial band jamming, assuming that the receiver not knowing ρ, uses the optimal T for the broadband jamming case.

4.5.1 Uniform Channel and Broadband Jammer

The basic receiver structure is that shown in figure 2 , but

the output of each energy detector is ouantized into one of four output levels. The discrete output of M such enrgy detectors feed the computing circuit, which computes the metric for each code word and finally makes a decision.

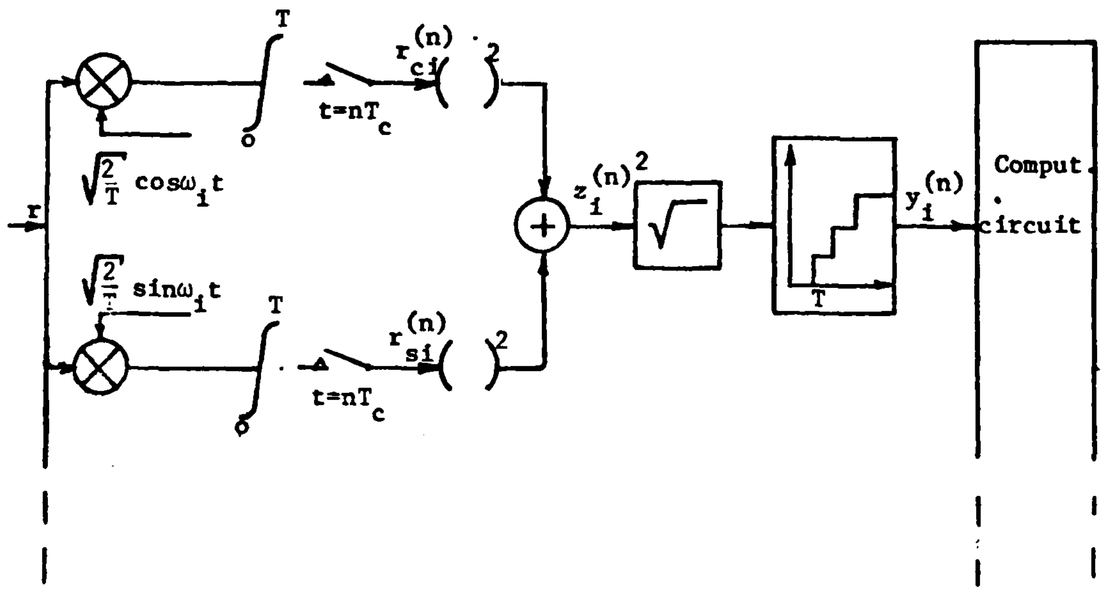


Figure 6 The Quantizer-Limiter Receiver

The input alphabet is $x \in \{1,2,\ldots,M\}$ and the output alphabet is:

$$\underline{y} = [y_1, y_2, \ldots, y_M]$$

where:

$$y_i \in \{0,1,2,3\} \quad ; \quad i=1,\ldots,M$$

The conditional probability of $\underline{y}$ given $\underline{x}$ is :

$$P_{mM}(\underline{y}/\underline{x}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)}) \tag{4.21}$$

where:

$$P_M(\underline{y}^{(n)}/x^{(n)}) = \left[\sum_{j=0}^{3} \delta\left(y_{x^{(n)}}^{(n)} - j\right) P_1^j\right] \prod_{\substack{k=1 \\ k \neq x^{(n)}}}^{M} \sum_{j=0}^{3} \delta\left(y_k^{(n)} - j\right) P_o^j$$

$$= \frac{\sum_{j=0}^{3} \delta\left(y_{x^{(n)}}^{(n)} - j\right) P_1^j}{\sum_{j=0}^{3} \delta\left(y_{x^{(n)}}^{(n)} - j\right) P_o^j} \prod_{k=1}^{M} \sum_{j=0}^{3} \delta(y_k^{(n)} - j) P_o^j \tag{4.22}$$

where :

$$P_o^j = P_r\{jT \leqslant Z_k^{(n)} < (j+1)T/x^{(n)} \neq k\} \quad ; \quad j=0,1,2 \; ; \; k=1,\ldots,M$$

$$P_o^3 = P_r\{3T \leqslant Z_k^{(n)}/x^{(n)} \neq k\} \quad ; \quad k=1,\ldots,M$$

$$n=1,\ldots,m$$

and :

$$P_1^j = P_r\{jT \leqslant Z_k^{(n)} < (j+1)T/x^{(n)} = k\} \quad ; \quad j=0,1,2; \quad k=1,\ldots,M$$

$$P_i^3 = P_r\{3T \leqslant Z_k^{(n)}/x^{(n)} = k\} \quad ; \quad k=1,\ldots,M$$

$$n=1,\ldots,m$$

To find P_o^j and P_1^j we note that $Z_k^{(n)}$ is Rayleigh distributed. In particular :

$$P(Z_k^{(n)}/x^{(n)}=k) = \frac{2Z_k^{(n)}}{N_J + \bar{E}_c} \exp\left\{-\frac{Z_k^{(n)^2}}{N_J + \bar{E}_c}\right\}$$

$$P(Z_k^{(n)}/x^{(n)} \neq k) = \frac{2Z_k^{(n)}}{N_J} \exp\left\{-\frac{Z_k^{(n)^2}}{N_J}\right\}$$

Hence :

$$P_o^o = 1 - \exp\left\{-\frac{T^2}{N_J}\right\}$$

$$P_o^1 = \exp\left\{-\frac{T^2}{N_J}\right\} - \exp\left\{-\frac{(2T)^2}{N_J}\right\}$$

$$P_o^2 = \exp\left\{-\frac{(2T)^2}{N_J}\right\} - \exp\left\{-\frac{(3T)^2}{N_J}\right\}$$

$$P_o^3 = \exp\left\{-\frac{(3T)^2}{N_J}\right\} \tag{4.23}$$

Whereas :

$$P_1^o = 1 - \exp\left\{-\frac{T^2}{N_J+\bar{E}_c}\right\}$$

$$P_1^1 = \exp\left\{-\frac{T^2}{N_J+\bar{E}_c}\right\} - \exp\left\{-\frac{(2T)^2}{N_J+\bar{E}_c}\right\}$$

$$P_1^2 = \exp\left\{-\frac{(2T)^2}{N_J+\bar{E}_c}\right\} - \exp\left\{-\frac{(3T)^2}{N_J}\right\}$$

$$P_1^3 = \exp\left\{-\frac{(3T)^2}{N_J+\bar{E}_c}\right\} \tag{4.24}$$

Hence :

$$P_{mM}(\underline{y}/\underline{x}) = \left[\prod_{n=1}^{m} \frac{\sum_{j=o}^{3} \delta(y_{x^{(n)}}^{(n)} - j) P_1^j}{\sum_{j=o}^{3} \delta(y_{x^{(n)}}^{(n)} - j) P_o^j}\right] G(\underline{y}) \tag{4.25}$$

where :

$$G(\underline{y}) = \prod_{n=1}^{m} \prod_{k=1}^{M} \sum_{j=o}^{3} \delta(y_k^{(n)} - j) P_o^j \tag{4.26}$$

Therefore, the ML receiver computes :

$$m(\underline{y};\underline{x}) = \ln \prod_{n=1}^{m} \frac{\sum_{j=0}^{3} \delta\left(y_{x^{(n)}}^{(n)} - j\right) P_1^j}{\sum_{j=0}^{3} \delta\left(y_{x^{(n)}}^{(n)} - j\right) P_0^j} \tag{4.27}$$

for every sequence $\underline{x}$ to determine the most likely sequence.

Since we are using the ML metric, the Chernoff bound reduces to the Bhattacharyya bound, and we obtain:

$$D(T) = \sum_{y_{x^{(n)}}^{(n)}} \sum_{y_{\hat{x}^{(n)}}^{(n)}} \sqrt{\left[\sum_{j=0}^{3} \delta\left(y_{x^{(n)}}^{(n)} - j\right) P_1^j\right]\left[\sum_{j=0}^{3} \delta\left(y_{\hat{x}^{(n)}}^{(n)} - j\right) P_0^j\right]} \cdot$$

$$\cdot \sqrt{\left[\sum_{j=0}^{3} \delta\left(y_{x^{(n)}}^{(n)} - j\right) P_0^j\right]\left[\sum_{j=0}^{3} \delta\left(y_{\hat{x}^{(n)}}^{(n)} - j\right) P_1^j\right]}$$

$$= \left[\sum_{y} \sqrt{\left[\sum_{j=0}^{3} \delta(y-j) P_1^j\right]\left[\sum_{j=0}^{3} \delta(y-j) P_0^j\right]}\right]^2$$

$$= \left[\sum_{j=0}^{3} \sqrt{P_1^j P_0^j}\right]^2 \tag{4.28}$$

Now let:

$$\Upsilon \triangleq \frac{T}{\sqrt{N_J}} \quad \text{and} \quad \Psi \triangleq \frac{\bar{E}_c}{N_J} \tag{4.29}$$

then :

$$\frac{T^2}{N_J} = \Upsilon^2 \quad \text{and} \quad \frac{T^2}{N_J + \bar{E}_c} = \frac{T^2}{N_J(1 + \bar{E}_c/N_J)} = \frac{\Upsilon^2}{1+\Psi}$$

Substituting the above in equation 4.28 we finally obtain:

$$D(\bar{T})=\left\{\sum_{j=0}^{2}\sqrt{\left[\exp\left\{-(j\bar{T})^2\right\}-\exp\left\{-[(j+1)\bar{T}]^2\right\}\right]\left[\exp\left\{-\frac{(j\bar{T})^2}{1+\Psi}\right\}-\exp\left\{-\frac{[(j+1)\bar{T}]^2}{1+\Psi}\right\}\right]}\right.$$
$$\left.+\exp\left\{-\frac{(3\bar{T})^2}{2}\left(1+\frac{1}{1+\Psi}\right)\right\}\right\}^2 \qquad (4.29)$$

Hence:

$$D = \min_{0<\bar{T}} D(\bar{T})$$

The minimizing value of $\bar{T}$ depends on Ψ, and therefore, $T = \bar{T} N_J$ depends on both N_J and E_c. Figure 7 shows the optimal value of $\bar{T}$ as a function of Ψ and the value of R_0 obtained with the minimizing $\bar{T}$ under broadband jamming.

4.5.2 The Quantizier-Limiter Receiver Under Partial Band Noise Jamming

We assume negligible background noise. When jammed the power spectral density of the noise is N_J/ρ where:

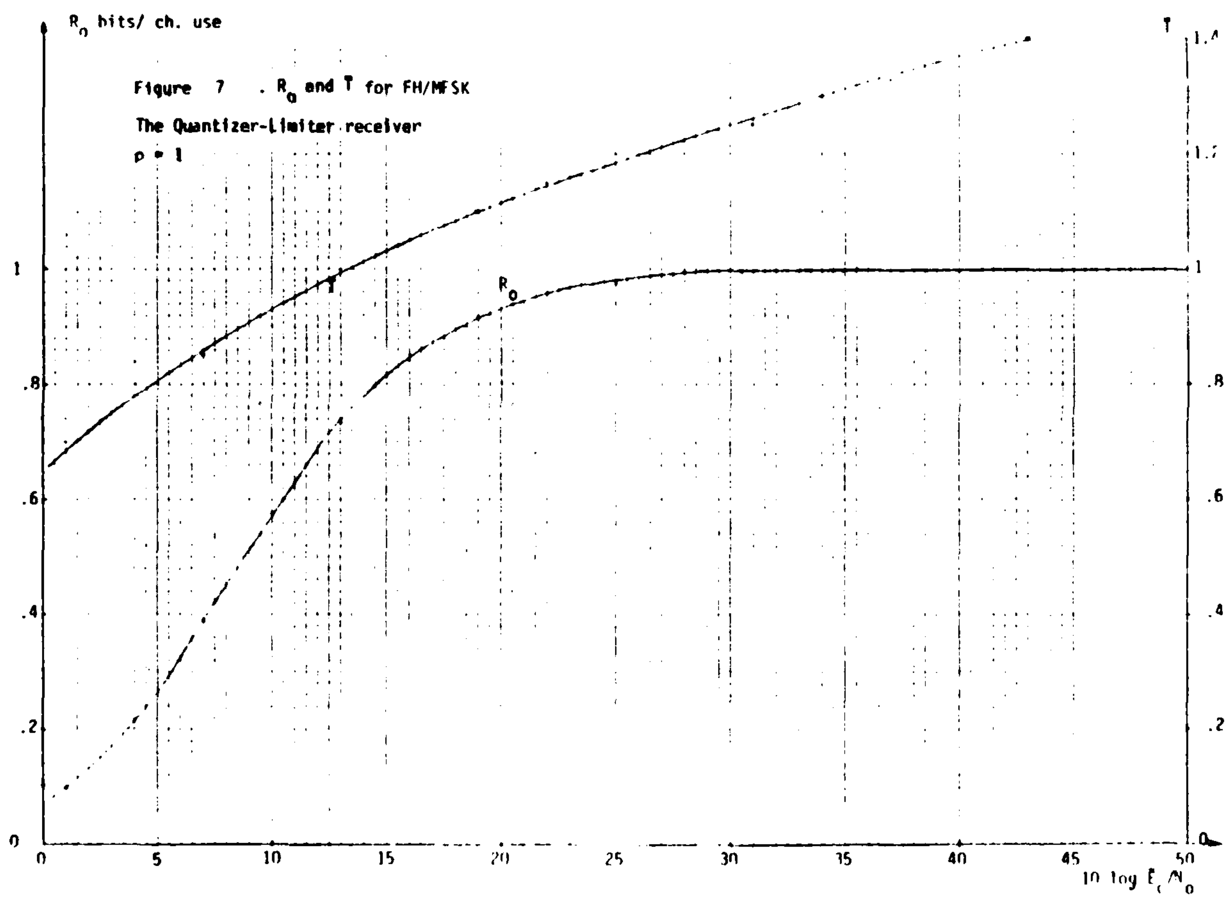
$$N_J = \frac{J}{W}$$

The receiver, not knowing ρ assumes continuous/ broadband jamming and therefore uses the same metric and the same T as for the $\rho = 1$ case:

$$m(\underline{y}^{(n)};x^{(n)}) = \ln \frac{\sum_{j=0}^{3} \delta\left(y^{(n)}_{x^{(n)}}-j\right)P_1^j}{\sum_{j=0}^{3} \delta\left(y^{(n)}_{x^{(n)}}-j\right)P_0^j} \qquad (4.30)$$

but now this is no longer the ML metric , hence:

$$P(\underline{x} \to \hat{\underline{x}}) \leq E\left[\exp\left(\lambda\,[m(\underline{y};\hat{\underline{x}})- m(\underline{y};\underline{x})]\right) / \underline{x}\right] =$$



R_0 bits/ ch. use
$\bar{T}$
Figure 7 . R_0 and $\bar{T}$ for FH/MFSK
The Quantizer-Limiter receiver
$\rho = 1$
R_0
$\bar{T}$
0
.2
.4
.6
.8
1
1.2
1.4
0
5
10
15
20
25
30
35
40
45
50
10 log E_c/N_0

$$= E\left[\exp\left\{\lambda\sum_{n=1}^{m}\left\{\ln\frac{\sum_{j=0}^{3}\delta\left(y^{(n)}_{\hat{x}^{(n)}}-j\right)P_1^j}{\sum_{j=0}^{3}\delta\left(y^{(n)}_{\hat{x}^{(n)}}-j\right)P_0^j} - \ln\frac{\sum_{j=0}^{3}\delta\left(y^{(n)}_{x^{(n)}}-j\right)P_1^j}{\sum_{j=0}^{3}\delta\left(y^{(n)}_{x^{(n)}}-j\right)P_0^1}\right\}\right\}\Big/\underline{x}\right]$$

For brevity let:

$$G(y) \triangleq \frac{\sum_{j=0}^{3}\delta(y-j)P_1^j}{\sum_{j=0}^{3}\delta(y-j)P_0^j} \tag{4.31}$$

Then : for $\hat{x}^{(n)} \neq x^{(n)}$:

$$D(x^{(n)},\hat{x}^{(n)};\lambda) = E\left[\exp\left\{\lambda\ln\frac{G\left(y^{(n)}_{\hat{x}^{(n)}}\right)}{G\left(y^{(n)}_{x^{(n)}}\right)}\right\}\Big/x^{(n)}\right]$$

$$= E\left\{\left[\frac{G\left(y^{(n)}_{\hat{x}^{(n)}}\right)}{G\left(y^{(n)}_{x^{(n)}}\right)}\right]^{\lambda}\Big/x^{(n)}\right\}$$

$$= \rho E\left\{\left[\frac{G\left(y^{(n)}_{\hat{x}^{(n)}}\right)}{G\left(y^{(n)}_{x^{(n)}}\right)}\right]^{\lambda}\Big/x^{(n)}, Z_n=1\right\} + (1-\rho)E\left\{\left[\frac{G\left(y^{(n)}_{\hat{x}^{(n)}}\right)}{G\left(y^{(n)}_{x^{(n)}}\right)}\right]^{\lambda}\Big/x^{(n)}, Z_n=0\right\}$$

Now we must define two sets of probabilities: one describing $\underline{y}^{(n)}$ when the jammer is present and the other when the jammer is absent.

Let:

$$P_0^j(i) \triangleq P_r\{y_k^{(n)}=j/x^{(n)}\neq k,\ Z_n=i\}$$

$$j=0,1,2,3\ ;\quad k=1,\dots,M$$

$$i=0,1,\quad n=1,\dots,m$$

$$P_1^j(i) \triangleq P_r\{y_k^{(n)}=j/x^{(n)}=k,\ Z_n=i\}$$

Then:

$$P_o^j(0) = \begin{cases} 1 & ; \quad j=0 \\ 0 & ; \quad j\neq 0 \end{cases}$$

$$P_1^j(0) = \exp\left\{-\frac{(jT)^2}{\bar{E}_c}\right\} - \exp\left\{-\frac{[(j+1)T]^2}{\bar{E}_c}\right\} \quad ; \quad j=0,1,2$$

$$P_1^3(0) = \exp\left\{-\frac{(3T)^2}{\bar{E}_c}\right\}$$

$$P_o^j(1) = \exp\left\{-\frac{(jT)^2}{N_J/\rho}\right\} - \exp\left\{-\frac{[(j+1)T]^2}{N_J/\rho}\right\} \quad ; \quad j=0,1,2$$

$$P_o^3(1) = \exp\left\{-\frac{(3T)^2}{N_J/\rho}\right\} \tag{4.32}$$

and:

$$P_1^j(1) = \exp\left\{-\frac{(jT)^2}{N_J/\rho+\bar{E}_c}\right\} - \exp\left\{-\frac{[(j+1)T]^2}{N_J/\rho+\bar{E}_c}\right\} \quad ; \quad j=0,1,2$$

$$P_1^3(1) = \exp\left\{-\frac{(3T)^2}{N_J/\rho+\bar{E}_c}\right\} \tag{4.33}$$

Hence:

$$D(x^{(n)},\hat{x}^{(n)};\lambda) =$$

$$=\rho \sum_{y_{x^{(n)}}^{(n)}} G\left(y_{x^{(n)}}^{(n)}\right)^{-\lambda}\left[\sum_{j=0}^{3}\delta\left(y_{x^{(n)}}^{(n)}-j\right)P_1^j(1)\right]\sum_{y_{\hat{x}^{(n)}}^{(n)}} G\left(y_{\hat{x}^{(n)}}^{(n)}\right)^{\lambda}\left[\sum_{j=0}^{3}\delta\left(y_{\hat{x}^{(n)}}^{(n)}-j\right)P_o^j(1)\right]+$$

$$+(1-\rho)\sum_{y_{x^{(n)}}^{(n)}} G\left(y_{x^{(n)}}^{(n)}\right)^{-\lambda}\left[\sum_{j=0}^{3}\delta\left(y_{x^{(n)}}^{(n)}-j\right)P_1^j(0)\right]\cdot$$

$$\cdot\sum_{y_{\hat{x}^{(n)}}^{(n)}} G\left(y_{\hat{x}^{(n)}}^{(n)}\right)^{\lambda}\left[\sum_{j=0}^{3}\delta\left(y_{\hat{x}^{(n)}}^{(n)}-j\right)P_o^j(0)\right]$$

$$=(1-\rho)\left(\frac{P_1^o}{P_o^o}\right)^\lambda \sum_{j=0}^{3} P_1^j(o)\left(\frac{P_o^j}{P_1^j}\right)^\lambda + \rho \sum_{j=0}^{3} P_1^j(1)\left(\frac{P_o^j}{P_1^j}\right)^\lambda \sum_{j=0}^{3} P_o^j(1)\left(\frac{P_1^j}{P_o^j}\right)^\lambda =$$

$$= D(\lambda) \qquad (4.34)$$

Where

$$D = \min_{o<\lambda} D(\lambda)$$

and:

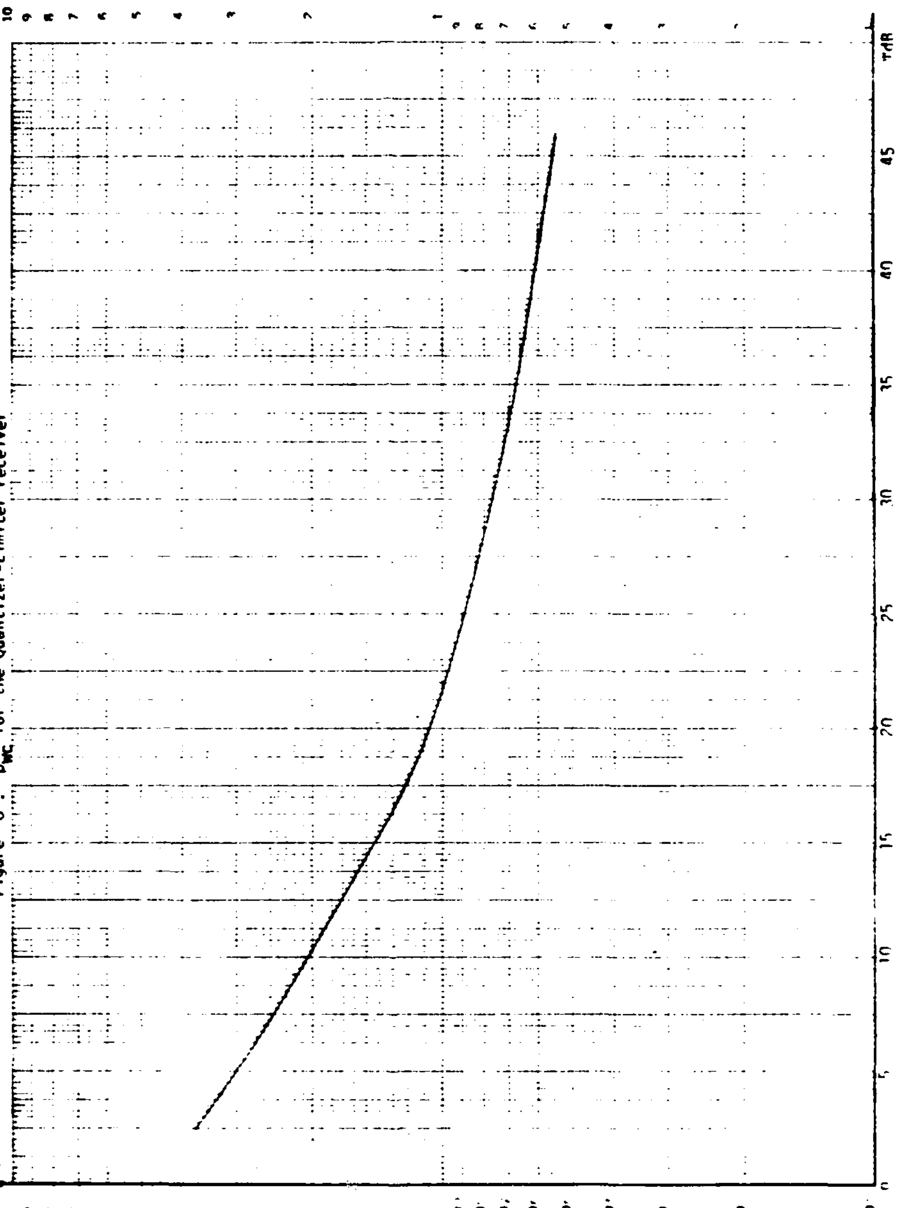
$$D_{wc} = \min_{o<\lambda} \max_{o<\rho\leq 1} D(\lambda) \qquad (4.35)$$

This bound is valid for any value of N_J, $\bar{E}_c$ and T, or ,using the normalized variables, for any value of Ψ and $\bar{T}$. It is of special interest , though, to evaluate the performance of the Quantizer-Limiter receiver when, for each value of Ψ, the receiver,not knowing ρ, selects that value of $\bar{T}$ which optimizes its performance under broadband jamming. That value of $\bar{T}$ is shown in figure 7 . Choosing $\bar{T}$ according to this figure, we have calculated the "worst case ρ" and the bound 4.35 as a function of Ψ. ρ_{wc} is shown in figure 8 and the bound 4.35 was used to compute R_o, as shown in figure 9 .Note that the performance of the Quantizer-Limiter receiver, as shown by the bound 4.35, is (up to Ψ=34 DB) better than that of the Hard Decision receiver.It could clearly be further enhanced,if the "truly" optimal $\bar{T}$ were used.

4.6 The Limiter Receiver With No JSI

This receiver has the same basic structure as the Soft Decision receiver discussed above. The only difference is that the output of each energy detector is clipped at $y_i = T$, i=1,...,M.

The outputs of the M clippers feed the computing circuit, which com-



K+E

Figure 8. P_{WC} for the Quantizer-Limiter receiver

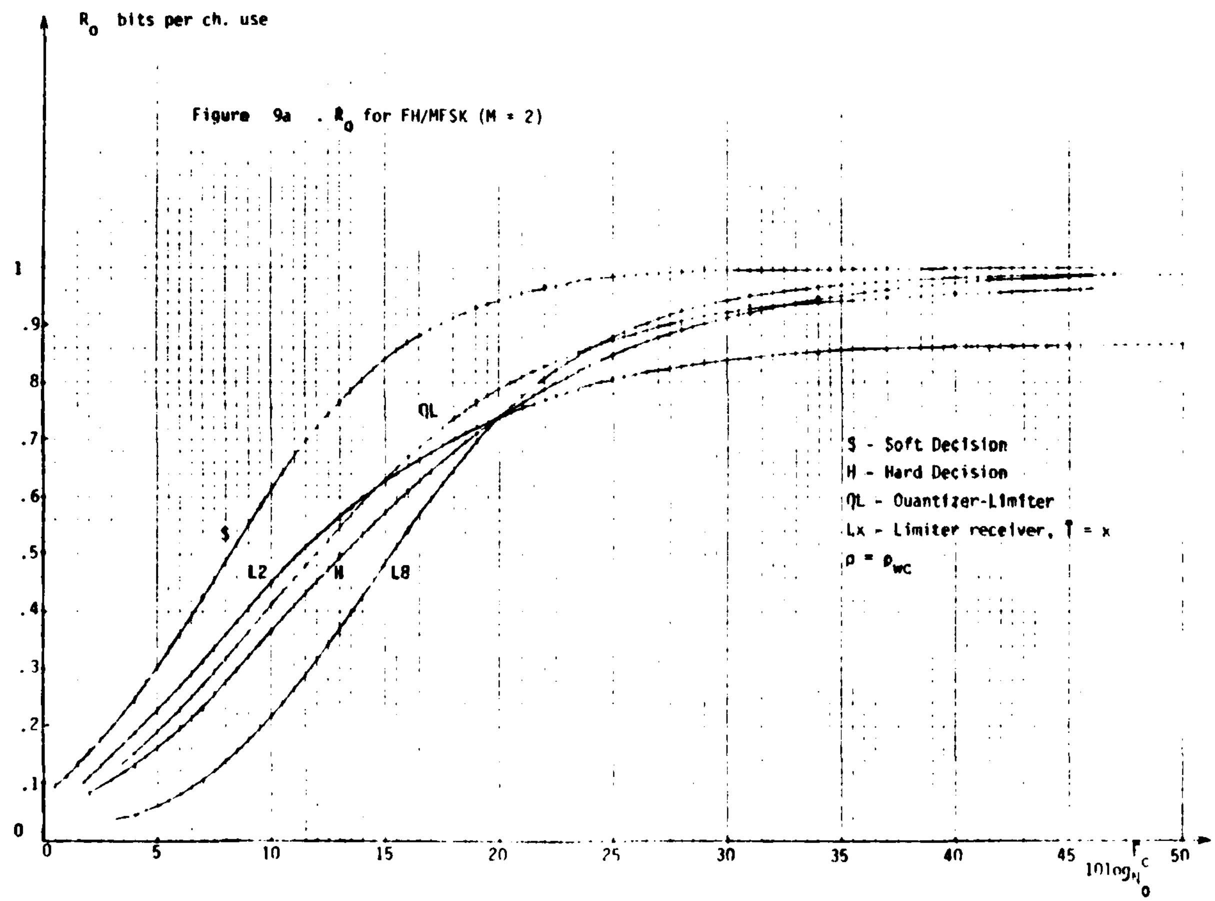


Figure 9a . R_0 for FH/MFSK (M = 2)

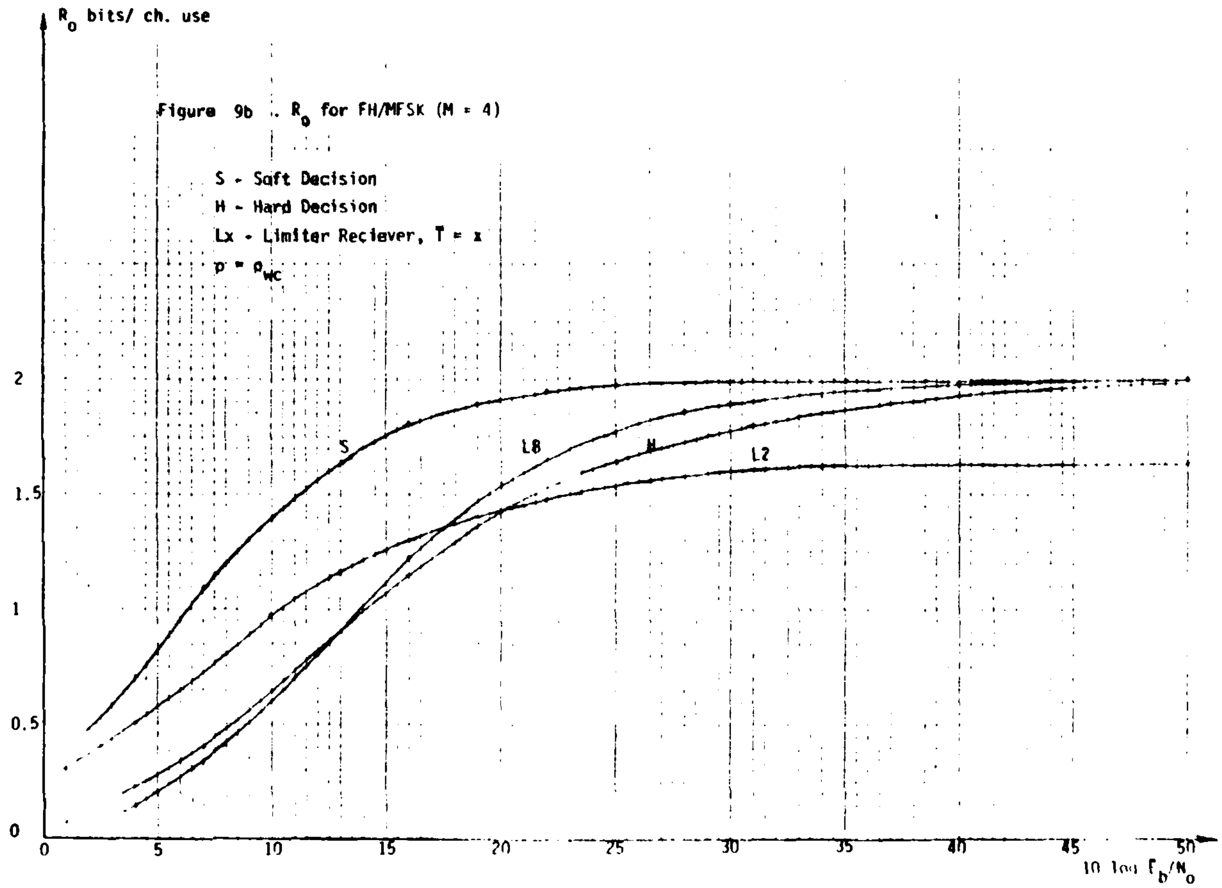


Figure 9b . R_o for FH/MFSK (M = 4)

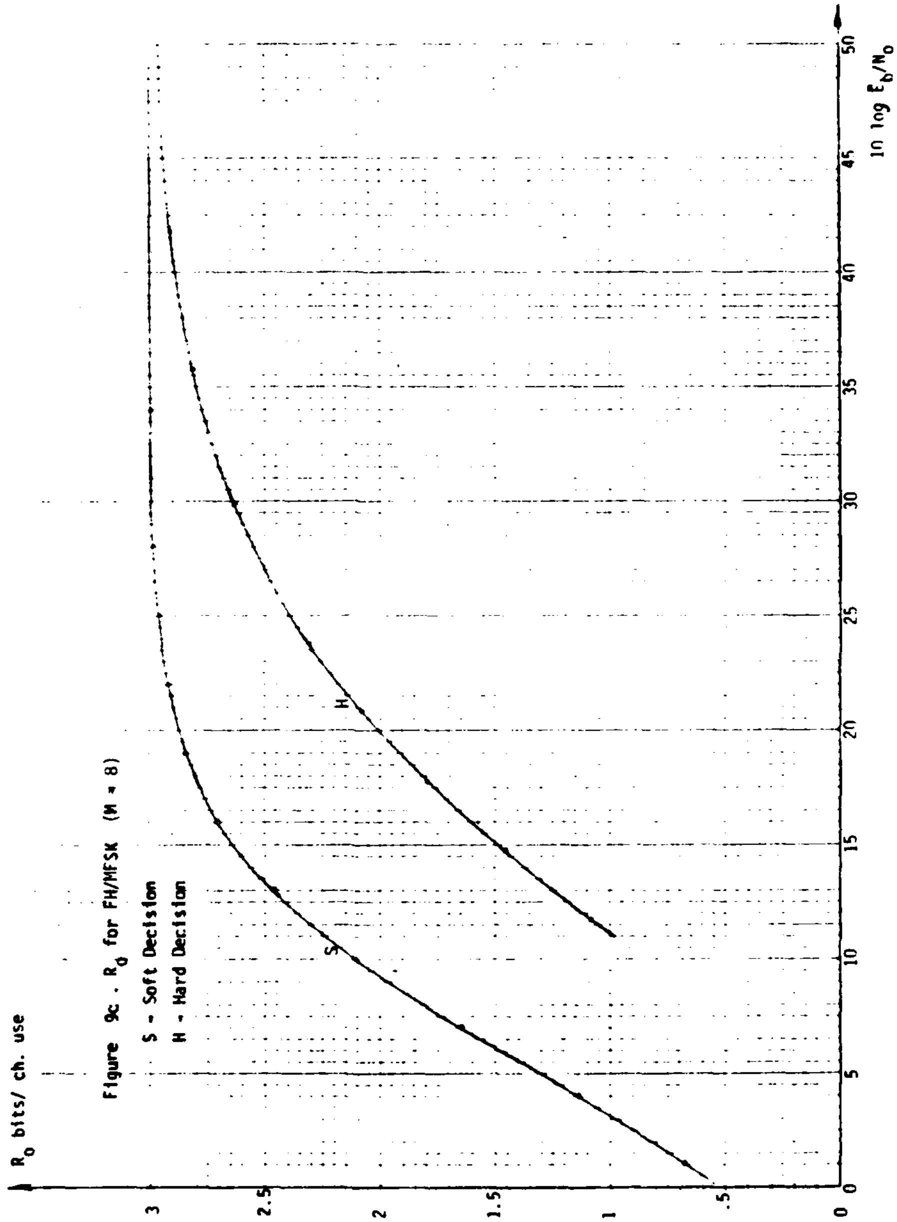


Figure 9c. R_0 for FH/MFSK (M = 8)

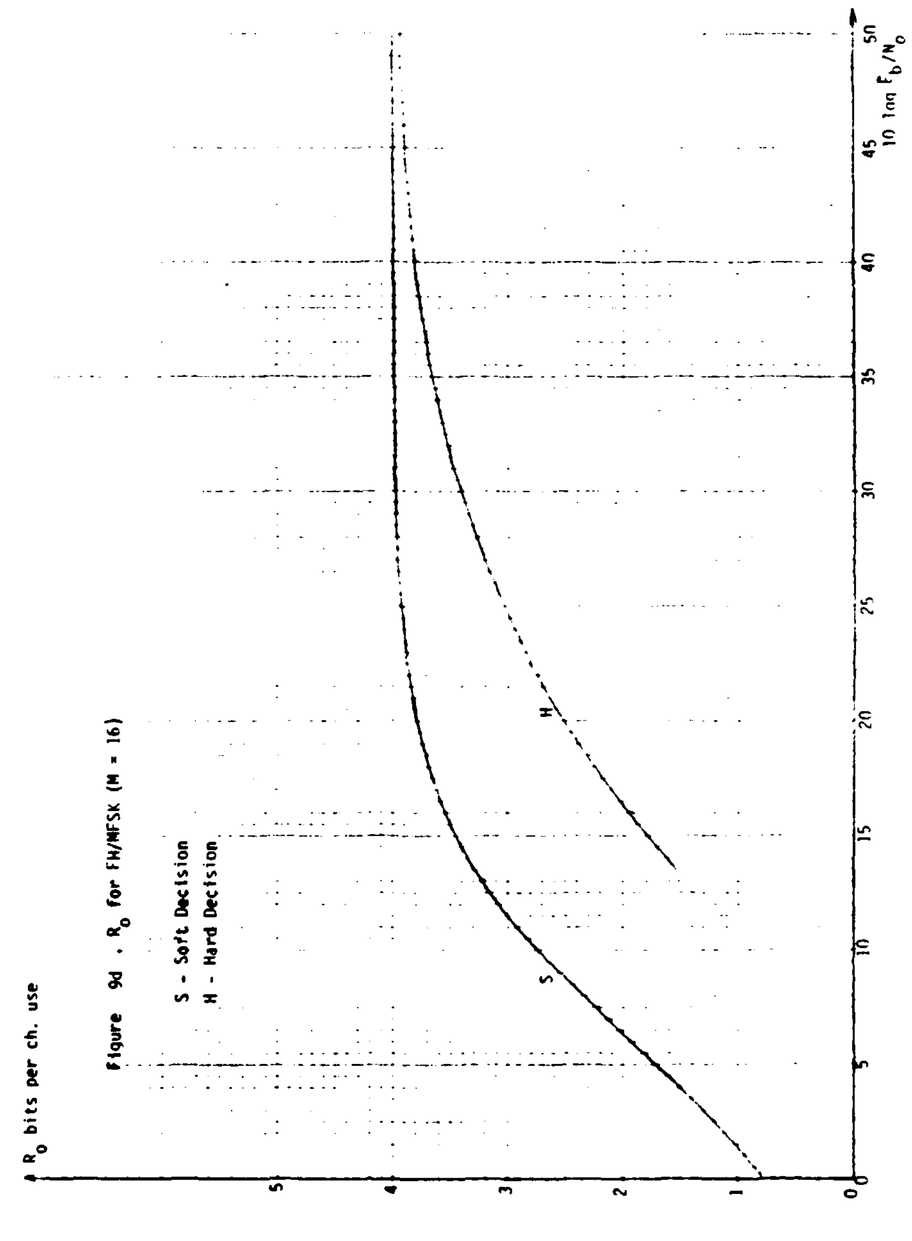


Figure 9d. R_o for FH/MFSK (M = 16)

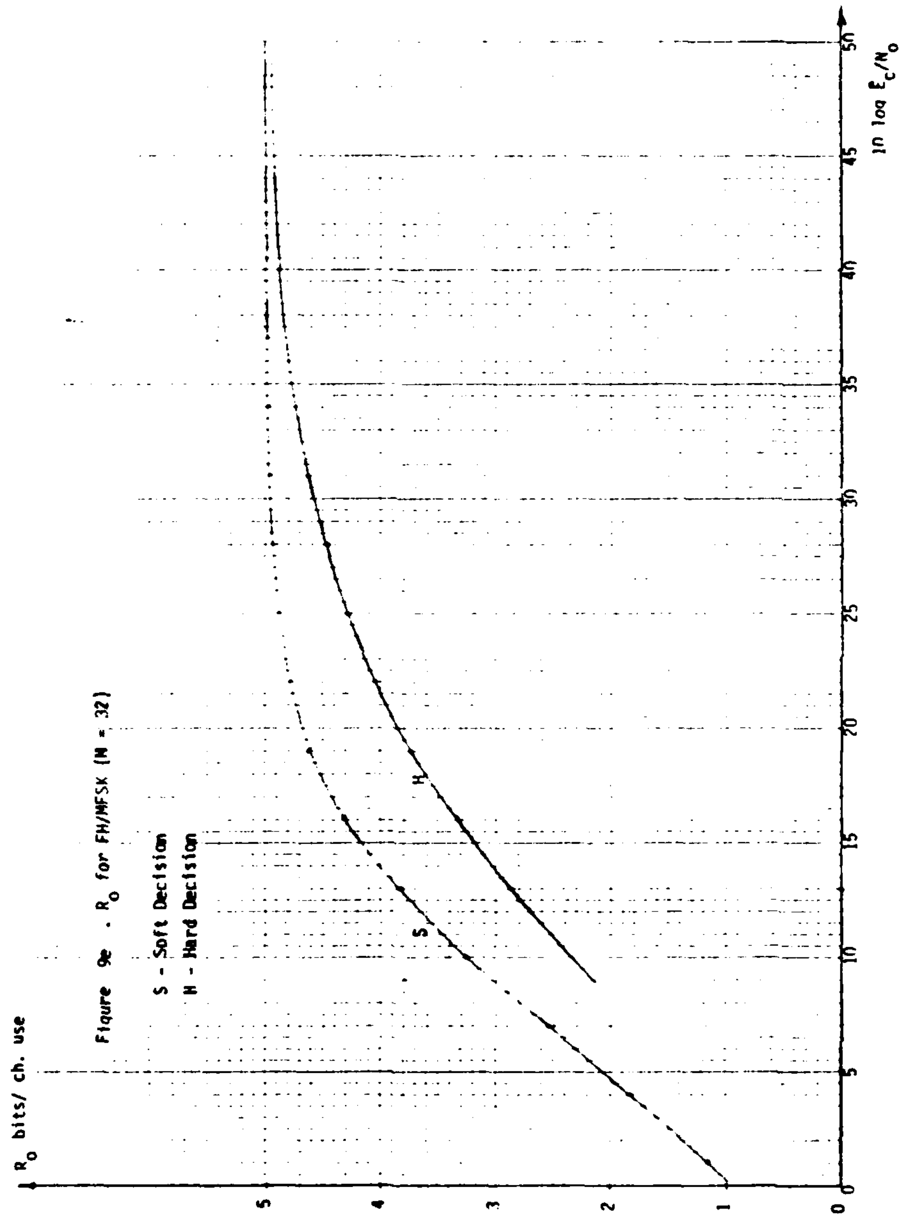


Figure 9e . R_o for FH/MFSK (M = 32)

S - Soft Decision
H - Hard Decision

putes the metric for each code word and finally makes a decision.

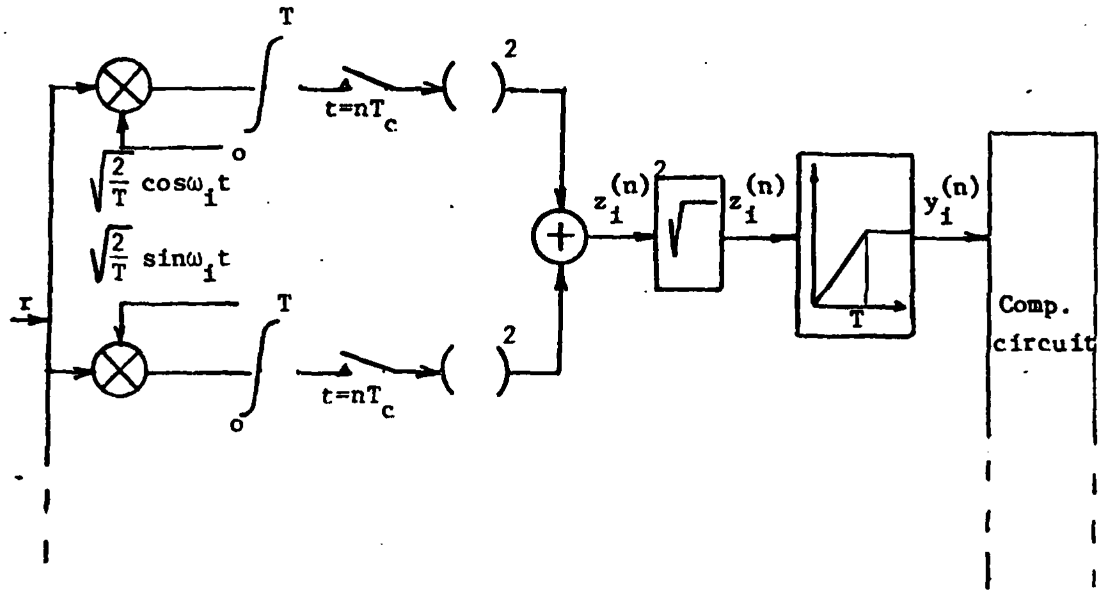


Figure 10 the Limiter receiver

4.6.1 Uniform Channel and Broadband Jamming

The input alphabet is : $x = \{1,2,\ldots,M\}$ and the output alphabet is: $\underline{y} = [y_1, y_2, \ldots, y_M]$ where: $y_i \in [0,T]$

The conditional probability of $\underline{y}$ given $\underline{x}$ is:

$$P_{mM}(\underline{y}/\underline{x}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)}) \tag{4.36}$$

Where:

$$P_M(\underline{y}^{(n)}/x^{(n)}) = Q_1(y_{x^{(n)}}^{(n)}) \prod_{\substack{k=1 \\ k\neq x^{(n)}}}^{M} Q_0(y_k^{(n)}) \tag{4.37}$$

Here:

$$Q_1(y) \triangleq \delta(y-T)P_1 + [1-u(y-T)]\frac{2y}{N_J+E_c} \exp\left\{-\frac{y^2}{N_J+E_c}\right\}; \quad y \geq 0 \tag{4.38}$$

where:

$$P_1 \triangleq P_r\{T \leq Z_k^{(n)}/x^{(n)}=k\} , \quad k=1,\ldots M ; \quad n=1,\ldots,m$$

$$= \exp\left\{-\frac{T^2}{N_J+E_c}\right\} \tag{4.39}$$

and $u(x)$ is the unit step function.

Similarly:

$$Q_o(y) \triangleq \delta(y-T)P_o + [1-u(y-T)]\frac{2y}{N_J} \exp\left\{-\frac{y^2}{N_J}\right\} \quad ; \quad y \geq 0 \tag{4.40}$$

where:

$$P_o \triangleq P_r\{T \leq Z_k^{(n)}/x^{(n)} \neq k\} , \quad k=1\ldots M \quad ; \quad n=1,\ldots,m$$

$$= \exp\left\{-\frac{T^2}{N_J}\right\} \tag{4.41}$$

$$\therefore \quad P_M(\underline{y}^{(n)}/x^{(n)}) = \frac{Q_1\left(y^{(n)}_{x^{(n)}}\right)}{Q_o\left(y^{(n)}_{x^{(n)}}\right)} \prod_{k=1}^{M} Q_o(y_k^{(n)})$$

$$= \frac{Q_1\left(y^{(n)}_{x^{(n)}}\right)}{Q_o\left(y^{(n)}_{x^{(n)}}\right)} G(\underline{y}^{(n)}) \tag{4.42}$$

where:

$$G(\underline{y}^{(n)}) \triangleq \prod_{k=1}^{M} Q_o(y_k^{(n)}) \tag{4.43}$$

The ML receiver computes the total metric:

$$m(\underline{y};\underline{x}) = \ln \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)})$$

for each sequence $\underline{x}$ to find the most likely sequence. But, since $G(\underline{y}^{(n)})$ does not depend on $\underline{x}$, it suffices to compute the metric:

$$\bar{m}(\underline{y}^{(n)};x^{(n)}) = \ln \frac{Q_1\left(y^{(n)}_{x^{(n)}}\right)}{Q_o\left(y^{(n)}_{x^{(n)}}\right)} \tag{4.44}$$

Since we are using the ML metric, the Chernoff bound reduces to the Bhattacharyya bound, and we obtain:

$$D(T) = \int_{y^{(n)}_{x^{(n)}}} \int_{y^{(n)}_{\hat{x}^{(n)}}} \sqrt{Q_1\left(y^{(n)}_{x^{(n)}}\right)Q_o\left(y^{(n)}_{x^{(n)}}\right)Q_1\left(y^{(n)}_{x^{(n)}}\right)Q_o\left(y^{(n)}_{x^{(n)}}\right)}\; dy^{(n)}_{x^{(n)}} dy^{(n)}_{x^{(n)}}$$

$$= \left\{ \int_o^{\infty} \sqrt{\delta(y-T)P_1 + [1-u(y-T)]\frac{2y}{N_J+E_c} \exp\left\{-\frac{y^2}{N_J+E_c}\right\}} \;\cdot \right.$$

$$\left. \cdot \sqrt{\delta(y-T)P_o + [1-u(y-T)]\frac{2y}{N_J} \exp\left\{-\frac{y^2}{N_J}\right\}}\; dy \right\}^2$$

$$= \left[\sqrt{P_oP_1} + \int_o^{T_-} \sqrt{\frac{(2y)^2}{N_J(N_J+E_c)} \exp\left\{-y^2\left(\frac{1}{N_J+E_c} + \frac{1}{N_J}\right)\right\}}\; dy\right]^2$$

But:

$$\int_o^T y \exp\left\{-\frac{y^2}{2}a\right\}dy = \frac{1}{a}\left[1 - \exp\left\{-\frac{T^2}{2}a\right\}\right]$$

Hence:

$$D(T) = \left\{ \sqrt{P_o P_1} + \frac{2\sqrt{N_J(N_J+\bar{E}_c)}}{2N_J+\bar{E}_c} \left[1 - \exp\left\{ -\frac{T^2}{2} \frac{2N_J+\bar{E}_c}{N_J(N_J+\bar{E}_c)} \right\} \right] \right\}^2$$

Now let: (4.45)

$$\bar{T} \triangleq \frac{T}{\sqrt{N_J}}$$

$$\Psi \triangleq \frac{\bar{E}_c}{N_J}$$

Then:

$$D(\bar{T}) = \left\{ \sqrt{P_o P_1} + \frac{2\sqrt{1+\Psi}}{2+\Psi} \left[1 - \exp\left\{ -\frac{\bar{T}^2}{2} \frac{2+\Psi}{1+\Psi} \right\} \right] \right\}^2$$

$$= \left\{ \exp\left\{ -\frac{\bar{T}^2}{2} \frac{2+\Psi}{1+\Psi} \right\} + \frac{2\sqrt{1+\Psi}}{2+\Psi} \left[1 - \exp\left\{ -\frac{\bar{T}^2}{2} \frac{2+\Psi}{1+\Psi} \right\} \right] \right\}^2 \quad (4.46)$$

Note, that when $\bar{T} \to \infty$, the bound reduces to:

$$D = \frac{4(1+\Psi)}{(2+\Psi)^2}$$

which is the result we had before for the Soft Decision receiver without clipping under broadband jamming.

To improve his performance the receiver should choose $\bar{T}$ so as to minimize $D(\bar{T})$. Hence:

$$D = \min_{o<\bar{T}} D(\bar{T}) \quad (4.47)$$

4.6.2 Uniform Channel and Pulsed / Partial Band Noise Jamming

We assume negligible channel noise. When jammed the power spectral density of the noise is N_J/ρ where:

$$N_J = \frac{J}{W}$$

The receiver, not knowing ρ, assumes continuous / broadband jamming and therefore uses the same metric as for the $\rho=1$ case:

$$m(\underline{y}^{(n)};x^{(n)}) = \ell n \frac{Q_1\left(y^{(n)}_{x^{(n)}}\right)}{Q_0\left(y^{(n)}_{x^{(n)}}\right)}$$

but now this is no longer the ML metric. Hence:

$$P(\underline{x} \to \underline{\hat{x}}) \leq E\left[\exp\left[\lambda\left[m(\underline{y};\underline{\hat{x}}) - m(\underline{y};\underline{x})\right]\right] / \underline{x}\right]$$

$$= E\left[\exp\left\{\lambda \sum_{n=1}^{m}\left[\ell n \frac{Q_1\left(y^{(n)}_{\hat{x}^{(n)}}\right)}{Q_0\left(y^{(n)}_{\hat{x}^{(n)}}\right)} - \ell n \frac{Q_1\left(y^{(n)}_{x^{(n)}}\right)}{Q_0\left(y^{(n)}_{x^{(n)}}\right)}\right]\right\} / \underline{x}\right]$$

Then, for $\hat{x}^{(n)} \neq x^{(n)}$:

$$D(x^{(n)},\hat{x}^{(n)};\lambda) = E\left[\exp\left\{\lambda \ell n \frac{Q_1\left(y^{(n)}_{\hat{x}^{(n)}}\right)Q_0\left(y^{(n)}_{x^{(n)}}\right)}{Q_0\left(y^{(n)}_{\hat{x}^{(n)}}\right)Q_1\left(y^{(n)}_{x^{(n)}}\right)}\right\} / x^{(n)}\right]$$

$$= E\left[\left(\frac{Q_1\left(y^{(n)}_{\hat{x}^{(n)}}\right)Q_0\left(y^{(n)}_{x^{(n)}}\right)}{Q_0\left(y^{(n)}_{\hat{x}^{(n)}}\right)Q_1\left(y^{(n)}_{x^{(n)}}\right)}\right)^{\lambda} / x^{(n)}\right]$$

$$= \rho E\left\{\left(\frac{Q_1\left(y^{(n)}_{\hat{x}^{(n)}}\right)Q_0\left(y^{(n)}_{x^{(n)}}\right)}{Q_0\left(y^{(n)}_{\hat{x}^{(n)}}\right)Q_1\left(y^{(n)}_{x^{(n)}}\right)}\right)^{\lambda} / x^{(n)}, Z_n=1\right\} +$$

$$+ (1-\rho)E\left\{\left(\frac{Q_1\left(y^{(n)}_{\hat{x}^{(n)}}\right)Q_0\left(y^{(n)}_{x^{(n)}}\right)}{Q_0\left(y^{(n)}_{\hat{x}^{(n)}}\right)Q_1\left(y^{(n)}_{x^{(n)}}\right)}\right)^{\lambda} / x^{(n)}, Z_n=0\right\}$$

We must now define two probability distributions, one describing $\underline{y}^{(n)}$ when the jammer is present and the other when the jammer is absent.

Let:

$$P_1(0) = P_r\{T \leq Z_k^{(n)} / x^{(n)} = k, \ \ _n = 0\}$$

$$P_1(1) = P_r\{T \leq Z_k^{(n)} / x^{(n)} = k, \ \ Z_n = 1\} \qquad k = 1, \ldots, M$$

$$P_o(1) = P_r\{T \leq Z_k^{(n)} / x^{(n)} \neq k, \ \ Z_n = 1\} \qquad n = 1, \ldots, m$$

$$P_o(0) = P_r\{T \leq Z_k^{(n)} / x^{(n)} \neq k, \ \ Z_n = 0\} = \delta(T) \tag{4.48}$$

Hence:

$$P_1(0) = \exp\left\{- \frac{T^2}{\bar{E}_c}\right\} = \exp\{- \frac{\bar{T}^2}{\Psi}\}$$

$$P_1(1) = \exp\left\{- \frac{T^2}{N_J/\rho + \bar{E}_c}\right\} = \exp\{- \frac{T^2}{\Psi + 1/\rho}\}$$

$$P_o(1) = \exp\left\{- \frac{T^2}{N_J/\rho}\right\} = \exp\{-\rho \bar{T}^2\}$$

$$P_o(0) = \delta(T) \tag{4.49}$$

Therefore:

$$\rho E\left\{ \left(\frac{Q_1\left(y^{(n)}_{\hat{x}(n)}\right) Q_o\left(y^{(n)}_{x(n)}\right)}{Q_o\left(y^{(n)}_{\hat{x}(n)}\right) Q_1\left(y^{(n)}_{x(n)}\right)} \right)^{\lambda} \Big/ x^{(n)}, \ Z_n = 1 \right\} =$$

$$= \rho \int_{y^{(n)}_{x(n)}} \left(\frac{Q_1\left(y^{(n)}_{x(n)}\right)}{Q_o\left(y^{(n)}_{x(n)}\right)} \right)^{-\lambda} \cdot$$

$$\cdot \left[\delta\left(y^{(n)}_{x(n)} - T\right) P_1(1) + [1 - u(y^{(n)}_{x(n)} - T)] \frac{2y^{(n)}_{x(n)}}{N_J/\rho + E_c} \exp\left\{ - \frac{{y^{(n)}_{x(n)}}^2}{N_J/\rho + E_c} \right\} \right] dy^{(n)}_{x(n)} \cdot$$

$$\cdot \int_{y^{(n)}_{\hat{x}(n)}} \left(\frac{Q_1\left(y^{(n)}_{\hat{x}(n)}\right)}{Q_o\left(y^{(n)}_{\hat{x}(n)}\right)} \right)^{\lambda} \cdot$$

$$\cdot \left[\delta\left(y^{(n)}_{\hat{x}(n)} - T\right) P_o(1) + [1 - u(y^{(n)}_{\hat{x}(n)} - T)] \frac{2y^{(n)}_{\hat{x}(n)}}{N_J/\rho} \exp\left\{ - \frac{y^{(n)2}_{\hat{x}(n)}}{N_J/\rho} \right\} \right] dy^{(n)}_{\hat{x}(n)}$$

$$= \left[\left(\frac{P_o}{P_1} \right)^{\lambda} P_1(1) + \int_o^T \left(\frac{N_J + \bar{E}_c}{N_J} \right)^{\lambda} \frac{2y}{N_J/\rho + \bar{E}_c} \exp\left\{ -y^2 \left(\frac{\lambda}{N_J} - \frac{\lambda}{N_J + \bar{E}_c} + \frac{1}{N_J/\rho + \bar{E}_c} \right) \right\} dy \right] \rho \cdot$$

$$\cdot \left[\left(\frac{P_1}{P_o} \right)^{\lambda} P_o(1) + \int_o^T \left(\frac{N_J}{N_J + \bar{E}_c} \right)^{\lambda} \frac{2y}{N_J/\rho} \exp\left\{ - y^2 \left(\frac{\lambda}{N_J + \bar{E}_c} - \frac{\lambda}{N_J} + \frac{1}{N_J/\rho} \right) \right\} dy \right]$$

$$= \rho \left\{ \left(\frac{P_o}{P_1} \right)^{\lambda} P_1(1) + \frac{(1+\Psi)^{1+\lambda}}{\lambda\Psi(1/\rho+\Psi)+1+\Psi} \left[1 - \exp\left\{ - T^2 \frac{\lambda\Psi(1/\rho+\Psi) + 1+\Psi}{(1+\Psi)(1/\rho+\Psi)} \right\} \right] \right\} \cdot$$

$$\cdot \left\{ \left(\frac{P_1}{P_o} \right)^{\lambda} P_o(1) + \frac{(1+\Psi)^{1-\lambda}}{-\lambda\Psi/\rho+1+\Psi} \left[1 - \exp\left\{ - T^2 \frac{-\lambda\Psi/\rho+1+\Psi}{(1+\Psi)/\rho} \right\} \right] \right\}$$

And:

$$(1-\rho) E \left\{ \left(\frac{Q_1\left(y^{(n)}_{\hat{x}(n)}\right) Q_o\left(y^{(n)}_{x(n)}\right)}{Q_o\left(y^{(n)}_{\hat{x}(n)}\right) Q_1\left(y^{(n)}_{x(n)}\right)} \right)^{\lambda} \Big/ x^{(n)},\ Z_n = 0 \right\} =$$

$$= (1-\rho) \int_{y^{(n)}_{x(n)}} \left(\frac{Q_o\left(y^{(n)}_{x(n)}\right)}{Q_1\left(y^{(n)}_{x(n)}\right)} \right)^{\lambda} \cdot$$

$$\cdot\left[\delta\left(y^{(n)}_{x^{(n)}} - T\right)P_1(o) + \left[1-u\left(y^{(n)}_{x^{(n)}} - T\right)\right] \frac{2y^{(n)}_{x^{(n)}}}{\bar{E}_c}\exp\left\{-\frac{{y^{(n)}_{x^{(n)}}}^2}{\bar{E}_c}\right\}\right] dy^{(n)}_{x^{(n)}} \cdot$$

$$\cdot \int_{y^{(n)}_{\hat{x}(n)}} \left(\frac{Q_1\left(y^{(n)}_{\hat{x}(n)}\right)}{Q_o\left(y^{(n)}_{\hat{x}(n)}\right)}\right)^\lambda \delta\left(y^{(n)}_{\hat{x}(n)}\right) dy^{(n)}_{\hat{x}(n)}$$

$$=(1-\rho)\left[\left(\frac{P_o}{P_1}\right)^\lambda P_1(o) + \int_o^{T_-}\left(\frac{N_J+\bar{E}_c}{N_J}\right)^\lambda \frac{2y}{\bar{E}_c}\exp\left\{-\frac{\lambda y^2}{N_J}+\frac{\lambda y^2}{N_J+\bar{E}_c} - \frac{y^2}{\bar{E}_c}\right\}dy\right]\left(\frac{N_J}{N_J+\bar{E}_c}\right)^\lambda \cdot$$

$$= (1-\rho)\left\{\left(\frac{P_o}{P_1}\right)^\lambda P_1(o) + \frac{(1+\Psi)^{\lambda+1}}{\lambda\Psi^2+1+\Psi}\left[1-\exp\left\{-\bar{T}^2\frac{\lambda\Psi^2+1+\Psi}{\Psi(1+\Psi)}\right\}\right]\right\}\left(\frac{1}{1+\Psi}\right)^\lambda$$

$$=(1-\rho)\left\{\left(1-\frac{(1+\Psi)^{1+\lambda}}{\lambda\Psi^2+1+\Psi}\right)\exp\left\{-\bar{T}^2\frac{\lambda\Psi^2+1+\Psi}{\Psi(1+\Psi)}\right\}+\frac{(1+\Psi)^{1+\lambda}}{\lambda\Psi^2+1+\Psi}\right\}\frac{1}{(1+\Psi)^\lambda}$$

Hence:

$$D(\bar{T};\lambda,\rho) =$$

$$=\rho\left[\exp\left\{-\bar{T}^2\frac{\lambda\Psi(\Psi+1/\rho)+1+\Psi}{(1+\Psi)(\Psi+1/\rho)}\right\}\left[1-\frac{(1+\Psi)^{1+\lambda}}{\lambda\Psi(\Psi+1/\rho)+1+\Psi}\right]+\frac{(1+\Psi)^{1+\lambda}}{\lambda\Psi(\Psi+1/\rho)+1+\Psi}\right]\cdot$$

$$\cdot\left[\exp\left\{-\bar{T}^2\frac{-\lambda\Psi+\rho(1+\Psi)}{1+\Psi}\right\}\left[1-\frac{(1+\Psi)^{1-\lambda}}{1+\Psi-\lambda\Psi/\rho}\right]+\frac{(1+\Psi)^{1-\lambda}}{1+\Psi-\lambda\Psi/\rho}\right]+$$

$$+(1-\rho)\left[\exp\left\{-\bar{T}^2\frac{\lambda\Psi^2+1+\Psi}{\Psi(1+\Psi)}\right\}\left[1-\frac{(1+\Psi)^{1+\lambda}}{\lambda\Psi^2+1+\Psi}\right]+\frac{(1+\Psi)^{1+\lambda}}{\lambda\Psi^2+1+\Psi}\right]\frac{1}{(1+\Psi)^\lambda} \tag{4.50}$$

Minimizing this bound over λ we have:

$$D(\bar{T};\rho) = \min_{o<\lambda} D(\bar{T};\lambda,\rho)$$

and

$$D_{wc}(\bar{T}) = \max_{o<\rho\leq 1}\ \min_{o<\lambda} D(\bar{T};\lambda,\rho) \tag{4.51}$$

To improve his performance the receiver should choose T so as to minimize the bound. The effective bound is then:

$$D_{wc} = \min_{o<\bar{T}} \max_{o<\rho\leq 1} \min_{o<\lambda} D(\bar{T};\lambda,\rho) \tag{4.52}$$

Figure 11 shows $D_{wc}(\bar{T})$ as a function of Ψ for several values of $\bar{T}$. Figures 9a and 9b show the cutoff rate R_o of this receiver for M=2,4 in comparison to R_o of other receivers. Figure 11a shows ρ_{wc} for three values of $\bar{T}$.

Figure 11. FH/MFSK , D_{wc} for the Limiter Receiver

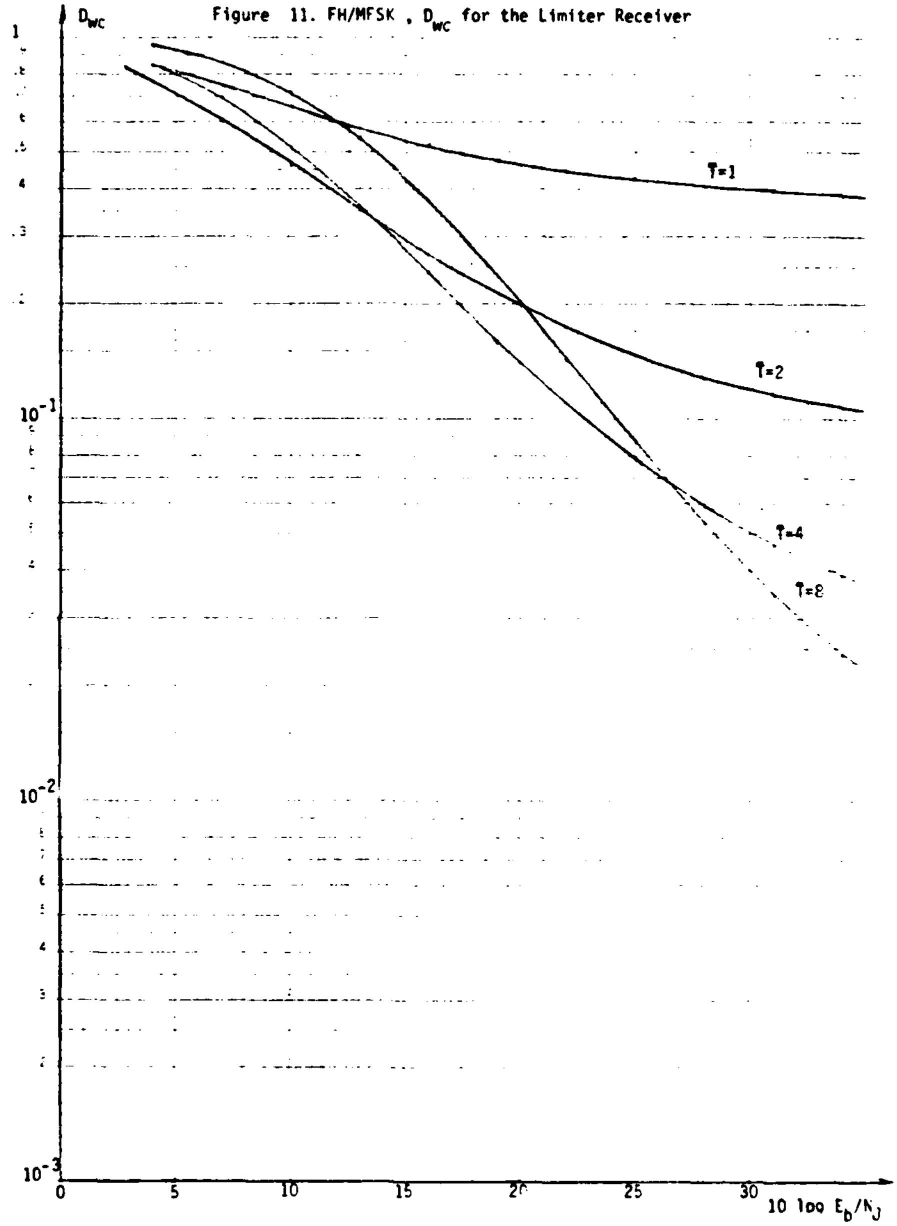
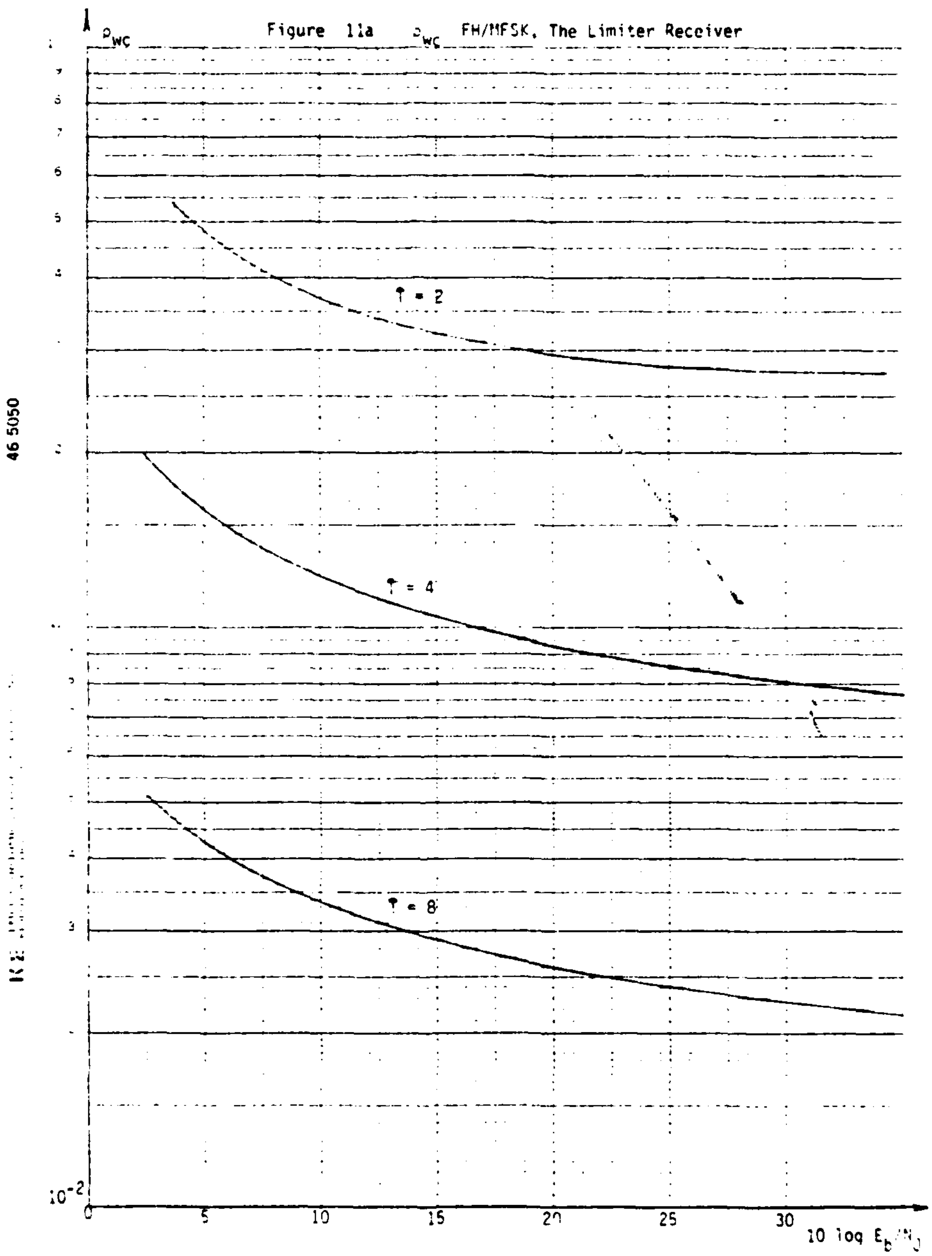


Figure 11a ρ_{wc} FH/MFSK, The Limiter Receiver



CHAPTER V

NON-UNIFORM CHANNELS

5. In this chapter we assume the following: The jammer uses pulsed noise (duty cycle ρ) and nonuniform distribution over the "Slotted Channel". At the j^{th} sub-band we have background noise of one-sided spectral density N_j, and in addition, jammer generated noise denoted N_{Jj}/ρ when the jammer is "on". As defined earlier, the hopping sequence is: $\underline{L} = (j_1, j_2, \ldots, j_m)$, where j_n is the index of the sub-band used in the n^{th} chip time, $j_n \in \{1,2,\ldots,N\}$. We assume that the random variables j_n, $n=1,\ldots,m$ are statistically independent discrete random variables having the common probability $P(j)$.

5.1 Soft Decision Receiver with JSI

The conditional density function of $\underline{y}$, given $\underline{x}$, $\underline{L}$ and $\underline{Z}$ is:

$$P_{mM}(\underline{y}/\underline{x},\underline{L},\underline{Z}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)},j_n,Z_n) \tag{5.1}$$

where

$$P_M(\underline{y}^{(n)}/x^{(n)},j_n,Z_n) = \prod_{k=1}^{M} P(y_k^{(n)}/x^{(n)},j_n,Z_n)$$

But

$$P(y_k^{(n)}/x^{(n)},j_n,Z_n=1) = \begin{cases} \dfrac{2y_k^{(n)}}{N_{j_n}+\bar{E}_{j_n}+N_{Jj_n}/\rho} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{j_n}+\bar{E}_{j_n}+N_{Jj_n}/\rho}\right\}; & x^{(n)}=k \\[2ex] \dfrac{2y_k^{(n)}}{N_{j_n}+N_{Jj_n}/\rho} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{j_n}+N_{Jj_n}/\rho}\right\} & ;\ x^{(n)}\neq k \end{cases}$$

and

$$P(y_k^{(n)}/x^{(n)}, j_n, Z_n=0) = \begin{cases} \dfrac{2y_k^{(n)}}{N_{j_n}+E_{j_n}} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{j_n}+E_{j_n}}\right\} & ; \quad x^{(n)}=k \\ \dfrac{2y_k^{(n)}}{N_{j_n}} \exp\left\{-\dfrac{y_k^{(n)^2}}{N_{j_n}}\right\} & ; \quad x^{(n)}\neq k \end{cases}$$

Next defining:

$$G(\underline{y},\underline{L},\underline{Z}) \triangleq \prod_{\substack{n=1\\ n:Z_n=1}}^{m} \frac{N_{j_n}+N_{Jj_n}/\rho}{N_{j_n}+E_{j_N}+N_{Jj_n}/\rho} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_{j_n}+N_{Jj_n}/\rho} \exp\left\{-\frac{y_k^{(n)^2}}{N_{j_n}+N_{Jj_n}/\rho}\right\}$$

$$\Delta_1(\underline{y};\underline{x},\underline{L},\underline{Z}) \triangleq \sum_{\substack{n=1\\ n:Z_n=1}}^{m} \frac{E_{j_n}}{\left(N_{j_n}+N_{Jj_n}/\rho\right)\left(N_{j_n}+E_{j_n}+N_{Jj_n}/\rho\right)} y_{x^{(n)}}^{(n)^2}$$

$$= \sum_{n=1}^{m} Z_n a_{j_n} y_{x^{(n)}}^{(n)^2}$$

Where

$$a_{j_n} \triangleq \frac{E_{j_n}}{\left(N_{j_n}+N_{Jj_n}/\rho\right)\left(N_{j_n}+E_{j_n}+N_{Jj_n}/\rho\right)} \qquad (5.2)$$

Also let

$$F(\underline{y},\underline{L},\underline{Z}) = \prod_{\substack{n=1\\ n:Z_n=0}}^{m} \frac{N_{j_n}}{N_{j_n}+E_{j_n}} \prod_{k=1}^{m} \frac{2y_k^{(n)}}{N_{j_n}} \exp\left\{-\frac{y_k^{(n)^2}}{N_{j_n}}\right\}$$

and

$$\Delta_o(\underline{y};\underline{x},\underline{L},\underline{Z}) = \sum_{\substack{n=1\\ n:Z_n=o}}^{m} \frac{E_{j_n}/N_{j_n}}{N_{j_n}+E_{j_n}} y_{x^{(n)}}^{(n)^2}$$

$$= \sum_{n=1}^{m} (1-Z_n) b_{j_n} y_{x^{(n)}}^{(n)^2}$$

where:

$$b_{j_n} \triangleq \frac{\bar{E}_{j_n}/N_{j_n}}{N_{j_n}+\bar{E}_{j_n}} \tag{5.3}$$

Then

$$P_{mM}(\underline{y}/\underline{x},\underline{L},\underline{Z}) = G(\underline{y},\underline{L},\underline{Z})\exp\{\Delta_1(\underline{y};\underline{x},\underline{L},\underline{Z})\}F(\underline{y},\underline{L},\underline{Z})\exp\{\Delta_0(\underline{y};\underline{x},\underline{L},\underline{Z})\}$$

The ML receiver uses the total metric:

$$m(\underline{y};\underline{x}/\underline{L},\underline{Z}) = \ln P_{mM}(\underline{y}/\underline{x},\underline{L},\underline{Z})$$

$$= \ln G(\underline{y},\underline{L},\underline{Z}) + \Delta_1(\underline{y};\underline{x},\underline{L},\underline{Z}) + \ln F(\underline{y},\underline{L},\underline{Z}) + \Delta_0(\underline{y},\underline{x},\underline{L},\underline{Z})$$

But, since $G(\underline{y},\underline{L},\underline{Z})$ and $F(\underline{y},\underline{L},\underline{Z})$ do not depend on $\underline{x}$, it suffices to compute

$$\bar{m}(\underline{y};\underline{x}/\underline{L},\underline{Z}) = \Delta_1(\underline{y};\underline{x},\underline{L},\underline{Z}) + \Delta_0(\underline{y};\underline{x},\underline{L},\underline{Z})$$

$$= \sum_{n=1}^{m}\left[Z_n a_{j_n} + (1-Z_n) b_{j_n}\right] y_{x^{(n)}}^{(n)^2} \tag{5.4}$$

for each sequence $\underline{x} \in C$ to determine the maximum likelihood sequence. Again we use the Bhattacharyya bound to compute the performance of this receiver.

$$P(\underline{x} \to \hat{\underline{x}}) \leq D^{W(\underline{x};\hat{\underline{x}})}$$

where

$$D = E\left\{\int_0^\infty\cdots\int_0^\infty \sqrt{P(\underline{y}^{(n)}/x^{(n)},j_n,Z_n)P(\underline{y}^{(n)}/\hat{x}^{(n)},j_n,Z_n)}\; dy_1^{(n)}\cdots dy_M^{(n)} \Big/ x^{(n)}\right\}$$

$$=(1-\rho)E\left\{\int_0^\infty\cdots\int_0^\infty \sqrt{P(\underline{y}^{(n)}/x^{(n)},j_n,0)P(\underline{y}^{(n)}/x^{(n)},j_n,0)}\; dy_1^{(n)}\cdots dy_M^{(n)} \Big/ x^{(n)}\right\}+$$

$$+\ \rho E\left\{\int_0^\infty\!\!\cdots\int_0^\infty \sqrt{P(\underline{y}^{(n)}/x^{(n)},j_n,1)P(\underline{y}^{(n)}/\hat{x}^{(n)},j_n,1)}\ dy_1^{(n)}\ldots dy_M^{(n)}\Big/x^{(n)}\right\}$$

Substituting the corresponding conditional probabilities and integrating over $y_i^{(n)}$, $i=1,\ldots,M$; $i\neq x^{(n)}$, $i\neq\hat{x}^{(n)}$

we are left with

$$D = (1-\rho)E\left\{\left[\int_0^\infty \frac{2y}{\sqrt{N_{j_n}\left(N_{j_n}+E_{j_n}\right)}}\exp\left\{-\frac{y^2}{2}\left(\frac{1}{N_{j_n}}+\frac{1}{N_{j_n}+E_{J_n}}\right)\right\}dy\right]^2\Big/x^{(n)}\right\}$$

$$+\rho E\left\{\left[\int_0^\infty \frac{2y\ dy}{\sqrt{\left(N_{j_n}+N_{Jj_n}/\rho\right)\left(N_{j_n}+E_{j_n}+N_{Jj_n}/\rho\right)}}\exp\left\{-\frac{y^2}{2}\left(\frac{1}{N_{j_n}+N_{Jj_n}/\rho}\right.\right.\right.\right.$$

$$\left.\left.\left.\left.+\frac{1}{N_{j_n}+E_{J_n}+N_{Jj_n}/\rho}\right)\right\}\right]^2\Big/x^{(n)}\right\}$$

$$= E\left\{(1-\rho)\frac{N_{j_n}\left(N_{j_n}+E_{j_n}\right)}{\left(N_{j_n}+E_{J_n}/2\right)^2}+\rho\frac{\left(N_{j_n}+N_{Jj_n}/\rho\right)\left(N_{j_n}+E_{j_n}+N_{Jj_n}/\rho\right)}{\left(N_{j_n}+E_{j_n}/2+N_{Jj_n}/\rho\right)^2}\Big/x^{(n)}\right\}$$

$$= \sum_{j=1}^{N} P(j)\left\{(1-\rho)\frac{4\left(1+\frac{E_j}{N_j}\right)}{\left(2+\frac{E_j}{N_j}\right)^2}+\frac{4\left(1+\frac{E_j}{N_j+N_{Jj}/\rho}\right)}{\left(2+\frac{E_j}{N_j+N_{Jj}/\rho}\right)^2}\rho\right\}$$

$$= \sum_{j=1}^{N} P(j) \left[\frac{4(1 + E_j/N_j)}{(2 + E_j/N_j)^2} + \frac{4E_j^2 N_{Jj}(2N_j + E_j + N_{Jj}/\rho)}{\left(2N_j + E_j\right)^2 \left(2N_j + E_j + 2N_{Jj}/\rho\right)^2} \right]$$

$$= \sum_{j=1}^{N} P(j) \left\{ \frac{4\left(1 + \frac{E_j}{N_j}\right)}{\left(2 + \frac{E_j}{N_j}\right)^2} + \rho \frac{4E_j^2 N_{Jj}}{\left(2N_j + E_j\right)^3} \frac{\left(\rho + \frac{N_{Jj}}{2N_j + E_j}\right)}{\left(\rho + \frac{2N_{Jj}}{2N_j + E_j}\right)^2} \right\} \tag{5.5}$$

It can be easily shown that $\rho=1$ maximizes this bound over the interval $0 < \rho \leq 1$. Hence, continuous jamming is the worst case jamming in this case also.

$$D_{WC} = \max_{0<\rho\leq 1} D = \sum_{j=1}^{N} P(j) \frac{4\left(1 + \frac{E_j}{N_j+N_{Jj}}\right)}{\left(2 + \frac{E_j}{N_j+N_{Jj}}\right)^2} \tag{5.6}$$

5.2 Soft Decision Receiver With No JSI

The jammer uses pulsed noise (duty cycle ρ) and nonuniform distribution over the Slotted Channel.

We have shown before that when the background noise is negligible and the channel is uniform, a receiver using the simple total metric:

$$m(\underline{y};\underline{x}) = \sum_{n=1}^{m} y_{x^{(n)}}^{(n)^2} \tag{5.7}$$

results in an unacceptable performance under low duty cycle jamming. Having the choice between this receiver and the Hard Decision receiver, the latter should of course be preferred. This is also the case when

the channel is not uniform and the background noise not negligible. Still, as discussed above for the special case, the receiver, not being able to detect the presence of the jammer for each chip time, may still be able to make a reasonable measurement of ρ. In this case the ML metric can be used. We follow this idea below.

The conditional probability of the channel is now

$$P_{mM}(\underline{y}/\underline{x},\underline{L}) = \prod_{n=1}^{m} P_M(\underline{y}^{(n)}/x^{(n)},j_n)$$

where

$$P_M(\underline{y}^{(n)}/x^{(n)},j_n) = \sum_{z=0}^{1} P_M(\underline{y}^{(n)}/x^{(n)},j_n,Z_n=z)P_{Z_n}(z)$$

$$= P_M(\underline{y}^{(n)}/x^{(n)},j_n,Z_n=0)(1-\rho) + P_M(\underline{y}^{(n)}/x^{(n)},j_n,Z_n=1)\rho$$

and

$$P_M(y_k^{(n)}/x^{(n)},j_n,Z_n=0) = \begin{cases} \dfrac{2y_k^{(n)}}{N_{j_n}+E_{j_n}} \exp\left\{-\dfrac{y_k^{(n)2}}{N_{j_n}+E_{j_n}}\right\} & ; \quad x^{(n)}=k \\[2ex] \dfrac{2y_k^{(n)}}{N_{j_n}} \exp\left\{-\dfrac{y_k^{(n)2}}{N_{j_n}}\right\} & ; \quad x^{(n)}\neq k \end{cases}$$

$$P_M(Y_k^{(n)}/x^{(n)},j_n,Z_n=1) = \begin{cases} \dfrac{2y_k^{(n)}}{N_{j_n}+E_{j_n}+N_{Jj_n}/\rho} \exp\left\{-\dfrac{y_k^{(n)2}}{N_{j_n}+E_{j_n}+N_{Jj_n}/\rho}\right\} & ; x^{(n)}=k \\[2ex] \dfrac{2y_k^{(n)}}{N_{j_n}+N_{Jj_n}/\rho} \exp\left\{-\dfrac{y_k^{(n)2}}{N_{j_n}+N_{Jj_n}/\rho}\right\} & ; x^{(n)}\neq k \end{cases}$$

Hence

$$P_{mM}(\underline{y}/\underline{x},\underline{L}) = \prod_{n=1}^{m}\left[(1-\rho)P_M(\underline{y}^{(n)}/x^{(n)},j_n,Z_n=0)+\rho P_M(\underline{y}^{(n)}/x^{(n)},j_n,Z_n=1)\right] \quad (5.8)$$

$$= \prod_{n=1}^{m} \Bigg[(1-\rho) \frac{2y_{x(n)}^{(n)}}{N_{j_n} + \bar{E}_{j_n}} \exp\left\{ - \frac{y_{x(n)}^{(n)2}}{N_{j_n} + \bar{E}_{j_n}} \right\} \prod_{\substack{k=1 \\ k \neq x(n)}}^{M} \frac{2y_k^{(n)}}{N_{j_n}} \exp\left\{ - \frac{y_k^{(n)2}}{N_{j_n}} \right\} +$$

$$+ \rho \frac{2y_{x(n)}^{(n)}}{N_{j_n} + \bar{E}_{j_n} + N_{Jj_n}/\rho} \exp\left\{ - \frac{y_{x(n)}^{(n)2}}{N_{j_n} + \bar{E}_{j_n} + N_{Jj_n}/\rho} \right\} \cdot$$

$$\cdot \prod_{\substack{k=1 \\ k \neq x(n)}}^{M} \frac{2y_k^{(n)}}{N_{j_n} + N_{Jj_n}/\rho} \exp\left\{ - \frac{y_k^{(n)2}}{N_{j_n} + N_{Jj_n}/\rho} \right\} \Bigg]$$

$$= \prod_{n=1}^{m} \Bigg[(1-\rho) \exp\left\{ \frac{y_{x(n)}^{(n)2} \bar{E}_{j_n}}{N_{j_n}(N_{j_n} + \bar{E}_{j_n})} \right\} G_{j_n}^{1}(\underline{y}^{(n)}) +$$

$$+ \rho \exp\left\{ \frac{y_{x(n)}^{(n)2} \bar{E}_{j_n}}{(N_{j_n} + N_{Jj_n}/\rho)(N_{j_n} + \bar{E}_{j_n} + N_{Jj_n}/\rho)} \right\} G_{j_n}^{2}(\underline{y}^{(n)}) \Bigg] \tag{5.9}$$

Where

$$G_{j_n}^{1}(\underline{y}^{(n)}) \triangleq \frac{N_{j_n}}{N_{j_n} + \bar{E}_{j_n}} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_{j_n}} \exp\left\{ - \frac{y_k^{(n)2}}{N_{j_n}} \right\}$$

and

$$G_{j_n}^{2}(y^{(n)}) \triangleq \frac{N_{j_n} + N_{Jj_n}/\rho}{N_{j_n} + \bar{E}_{j_n} + N_{Jj_n}/\rho} \prod_{k=1}^{M} \frac{2y_k^{(n)}}{N_{j_n} + N_{Jj_n}/\rho} \exp\left\{ - \frac{y_k^{(n)2}}{N_{j_n} + N_{Jj_n}/\rho} \right\}$$

To use the ML metric we take:

$$m(\underline{y};\underline{x}/\underline{L}) = \ln P_{mM}(\underline{y}/\underline{x},\underline{L}) =$$

$$= \sum_{n=1}^{m} \ell n \left[(1-\rho) G^1_{j_n}(\underline{y}^{(n)}) \exp\left\{ \frac{y^{(n)^2}_{x^{(n)}} E_{j_n}}{N_{j_n}(N_{j_n} + E_{j_n})} \right\} + \right.$$

$$\left. + \rho G^2_{j_n}(\underline{y}^{(n)}) \exp\left\{ \frac{y^{(n)^2}_{x^{(n)}} E_{j_n}}{(N_{j_n} + E_{j_n} + N_{Jj_n}/\rho)(N_{j_n} + N_{Jj_n}/\rho)} \right\} \right] \quad (5.10)$$

Obviously, this can hardly be considered a practical metric for a receiver. Nevertheless we proceed, trying to find $P(\underline{x} \rightarrow \hat{\underline{x}})$.

Since the receiver uses a ML metric, we can use the Bhattacharyya bound:

$$D = E\left\{ \int_0^\infty \cdots \int_0^\infty \sqrt{P_M(\underline{y}^{(n)}/x^{(n)}, j_n) P_M(\underline{y}^{(n)}/\hat{x}^{(n)}, j_n)}\, dy_1^{(n)} \cdots dy_M^{(n)} \Big/ x^{(n)} \right\} =$$

$$= \sum_{j=1}^{N} P(j) \int_0^\infty \cdots \int_0^\infty \sqrt{P_M(\underline{y}^{(n)}/x^{(n)}, j) P_M(\underline{y}^{(n)}/\hat{x}^{(n)}, j)}\, dy_1^{(n)} \cdots dy_M^{(n)} \quad (5.11)$$

and

$$D_{WC} = \max_{0<\rho\leq 1} D$$

It is difficult to proceed analytically to compute D, but, it can be easily verified that

$$\lim_{\rho \to 0} D(\rho) = \sum_{j=1}^{N} P(j) \frac{(1 + E_j/N_j)}{(1 + E_j/2N_j)^2}$$

which is exactly what we have with no jammer, i.e.,the jammer has no effect at all. For $\rho=1$, we obtain :

$$D(1) = \sum_{j=1}^{N} P(j) \frac{1 + \frac{\bar{E}_j}{N_j + N_{Jj}}}{\left(1 + \frac{\bar{E}_j}{2(N_j+N_{Jj})}\right)^2}$$

Which is what we obtained before for the receiver having JSI. This is what we would expect, since when $\rho=1$ both receivers are the same. However, for intermediate values of ρ, i.e., $0<\rho<1$, the performance differs.

Note that for the same receiver over a uniform channel, with no background noise, we have found that $\rho=1$ generates the worst case jamming. The same result seems to hold in the general case also.

5.3 Hard Decision Receiver with JSI

The jammer uses pulsed noise (duty cycle ρ) and nonuniform distribution over the Slotted Channel. The input and output alphabets of the channel are:

$$X = Y \in \{1,2,\ldots,M\}$$

and the conditional probability, when using the j^{th} sub-band ; $j=1,\ldots.,N$:

$$P(y/x,j,Z) = \begin{cases} 1-\varepsilon_j & ; \quad y=x \quad , \quad Z=0 \\ \frac{\varepsilon_j}{M-1} & ; \quad y\neq x \quad , \quad Z=0 \\ 1-\varepsilon_{Jj} & ; \quad y=x \quad , \quad Z=1 \\ \frac{\varepsilon_{Jj}}{M-1} & ; \quad y\neq x \quad , \quad Z=1 \end{cases} \qquad (5.12)$$

Where

$$\varepsilon_{Jj}(\rho) \triangleq \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1 + k\left(1 + \frac{E_j}{N_j + N_{Jj/\rho}}\right)} \tag{5.13}$$

$$\varepsilon_j = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1 + k\left(1 + \frac{E_j}{N_j}\right)} \tag{5.14}$$

The conditional probability of the channel is

$$P_m(\underline{y}/\underline{x},\underline{L},\underline{Z}) = \prod_{n=1}^{m} P(y_n/x_n,j_n,Z_n)$$

Using the ML metric

$$m(y_n;x_n/j_n,Z_n) = \ln P(y_n/x_n,j_n,Z_n)$$

We obtain the Bhattacharyya bound

$$P(\underline{x} \to \underline{\hat{x}}) \leq D^{W(\underline{x};\underline{x})}$$

where

$$D = E\left\{ \sum_{y_n} \sqrt{P(y_n/x_n,j_n,Z_n)P(y_n/\hat{x}_n,j_n,Z_n)} \Big/ \begin{matrix} x_n \\ \hat{x}_n \neq x_n \end{matrix} \right\} =$$

$$= \sum_{j=1}^{N} P(j) \sum_{z=o}^{1} P_{Z_n}(z) \sum_{y} \sqrt{P(y/x_n,j,z)P(y/\hat{x}_n,j,z)}$$

$$= \sum_{j=o}^{N} P(j) \left\{ (1-\rho) \left[2\sqrt{\frac{\varepsilon_j(1-\varepsilon_j)}{M-1}} + \frac{M-2}{M-1}\varepsilon_j \right] + \right.$$

$$+ \rho\left[2\sqrt{\frac{\varepsilon_{Jj}(1-\varepsilon_{Jj})}{M-1}} + \frac{M-2}{M-1}\varepsilon_{Jj}\right]\Bigg\} \tag{5.15}$$

Here also $\rho = 1$ maximizes D

$$\therefore \quad D_{Wc} \triangleq \max_{0<\rho\leq 1} D = \sum_{j=1}^{N} P(j)\left[2\sqrt{\frac{\varepsilon_{Jj}(1)[1-\varepsilon_{Jj}(1)]}{M-1}} + \frac{M-2}{M-1}\varepsilon_{Jj}(1)\right] \tag{5.16}$$

5.4 Hard Decision Receiver with No JSI

The jammer uses pulsed noise (duty cycle ρ) and nonuniform distribution over the "Slotted Channel".

The input and output alphabets are

$$X = Y \in \{1,2,\ldots,M\}$$

The conditional probability function, when using the j^{th} subchannel, $j = 1,\ldots,N$, is:

$$P(y/x,j) = \begin{cases} 1-\varepsilon_j(1-\rho)-\varepsilon_{Jj}\rho & ; \quad y=x \\ \\ \dfrac{\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho}{M-1} & ; \quad y\neq x \end{cases} \tag{5.17}$$

where

$$\varepsilon_j \triangleq \sum_{k=1}^{M-1}\binom{M-1}{k}(-1)^{k+1}\frac{1}{1+k(1+E_j/N_j)}$$

$$\varepsilon_{Jj}(\rho) \triangleq \sum_{k=1}^{M-1}\binom{M-1}{k}(-1)^{k+1}\frac{1}{1+k\left(1+\dfrac{E_j}{N_j+N_{Jj}/\rho}\right)}$$

The metric we use is again the ML metric :

$$m(\underline{y};\underline{x}/\underline{L}) = \ln P(\underline{y}/\underline{x},\underline{L})$$

Hence, we have the Bhattacharyya bound

$$P(\underline{x} \to \hat{\underline{x}}) \leq D^{W(\underline{x};\hat{\underline{x}})}$$

Where

$$D = E\left\{\sum_{y_n}\sqrt{P(y_n/x_n,j_n)P(y_n/\hat{x}_n,j_n)}\,/x^{(n)}\right\} \quad ; \quad x_n \neq \hat{x}_n$$

$$= \sum_{j=1}^{N} P(j) \sum_{y}\sqrt{P(y/x_n,j)P(y/\hat{x}_n,j)} =$$

$$= \sum_{j=1}^{N} P(j)\left[2\sqrt{\frac{[\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho][1-\varepsilon_j(1-\rho)-\varepsilon_{Jj}\rho]}{M-1}} + \right.$$

$$\left. + \frac{M-2}{M-1}\left(\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho\right)\right] \tag{5.18}$$

But

$$\max_{0<\rho\leq 1}\{\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho\} = \left|\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho\right|_{\rho=1} = \varepsilon_{Jj}(1)$$

It is therefore clear that

$$D_{Wc} = \max_{0<\rho\leq 1} D = \sum_{j=1}^{N} P(j)\left[2\sqrt{\frac{\varepsilon_{Jj}(1-\varepsilon_{Jj})}{M-1}} + \frac{M-2}{M-1}\varepsilon_{Jj}\right] \tag{5.19}$$

where

$$\varepsilon_{Jj} \triangleq \varepsilon_{Jj}(1)$$

Note, that if the receiver had no Channel State Information then

$$\bar{P}(\underline{y}/\underline{x}) = \begin{cases} 1-p \triangleq \sum_j P(j)[1-\varepsilon_j(1-\rho)-\varepsilon_{Jj}\rho] & ; \quad y=x \\ \frac{p}{M-1} \triangleq \sum_j P(j)\left[\frac{\varepsilon_j(1-\rho)+\varepsilon_{Jj}\rho}{M-1}\right] & ; \quad y\neq x \end{cases} \tag{5.20}$$

and using the ML metric, we would have obtained :

$$\bar{D} = 2\sqrt{\frac{p(1-p)}{M-1}} + \frac{M-2}{M-1}\,p \tag{5.21}$$

when the bar is used to discriminate between the two receivers. Since $D(\underline{P})$ is convex $\cap$, it is clear that $\bar{D} \geqslant D$.

CHAPTER VI

SIMPLE APPLICATIONS OF THE GENERAL BOUND

6. As shown in Chapter III, the cutoff rate for a coded bit in the worst case jamming environment is given by

$$R_o = \log_2[1+(M-1)D_{wc}] \quad \text{bits/channel use} \quad (6.1)$$

It is now a trivial matter to derive R_o for all the situations analyzed above. In particular, we have derived D_{wc} for the Soft Decision receiver with JSI over a uniform channel. Since $\rho = 1$ is the worst case jamming, we now let N_o represent the total uniform noise spectral density, which includes the background noise and the effect of the jamming. D_{wc} is in this case given by

$$D_{wc} = \frac{4\left[1 + \frac{\bar{E}_c}{N_o}\right]}{\left[2 + \frac{\bar{E}_c}{N_o}\right]^2} \quad (6.2)$$

Using eauations 6.1, 6.2,we have computed R_o for $M = 2,4,8,16,32$ as a function of $\bar{E}_b/N_o$. These results are shown in figure 9.

Since the Soft Decision receiver having knowledge of ρ only, achieves the same performance for $\rho = 1$, as the receiver having JSI. The same figure shows also the cutoff rate of the two Hard Decision receivers considered above. For both receivers over the uniform channel D_{Wc} is given by

$$D_{Wc} = 2\sqrt{\frac{\varepsilon(1-\varepsilon)}{M-1}} + \frac{M-2}{M-1}\,\varepsilon \quad (6.3)$$

where :

$$\varepsilon = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+E_c/N_o\right)} \tag{6.4}$$

In general, D_{Wc} is a function of E_c, $\underline{J}$, $\underline{N}$ & $\underline{P}$, but, for a uniform channel D_{Wc} can be written as a function of E_c/N_o only. To emphasize this fact we write :

$$D = D\left(\frac{E_c}{N_o}\right)$$

and:

$$R_o = R_o\left(\frac{E_c}{N_o}\right) \tag{6.5}$$

6.1 MFSK

Conventional MFSK modulation has the symbol error probability bound

$$P_S \leqslant \frac{1}{2}(M-1)D \tag{6.6}$$

and bit error bound

$$P_b = \frac{M/2}{M-1} P_S \tag{6.7}$$

Now, since for MFSK

$$E_c = KE_b , \tag{6.8}$$

where

$$K = \log_2 M \tag{6.9}$$

we have for the uniform channel

$$P_b \leqslant 2^{K-2} D\left(\frac{KE_b}{N_o}\right) \tag{6.10}$$

Hence, the bound of the Soft Decision receiver is in this case

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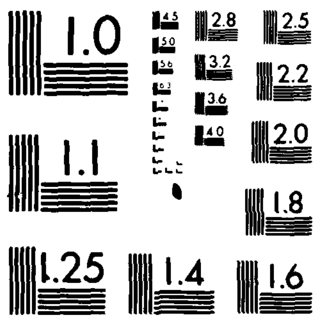

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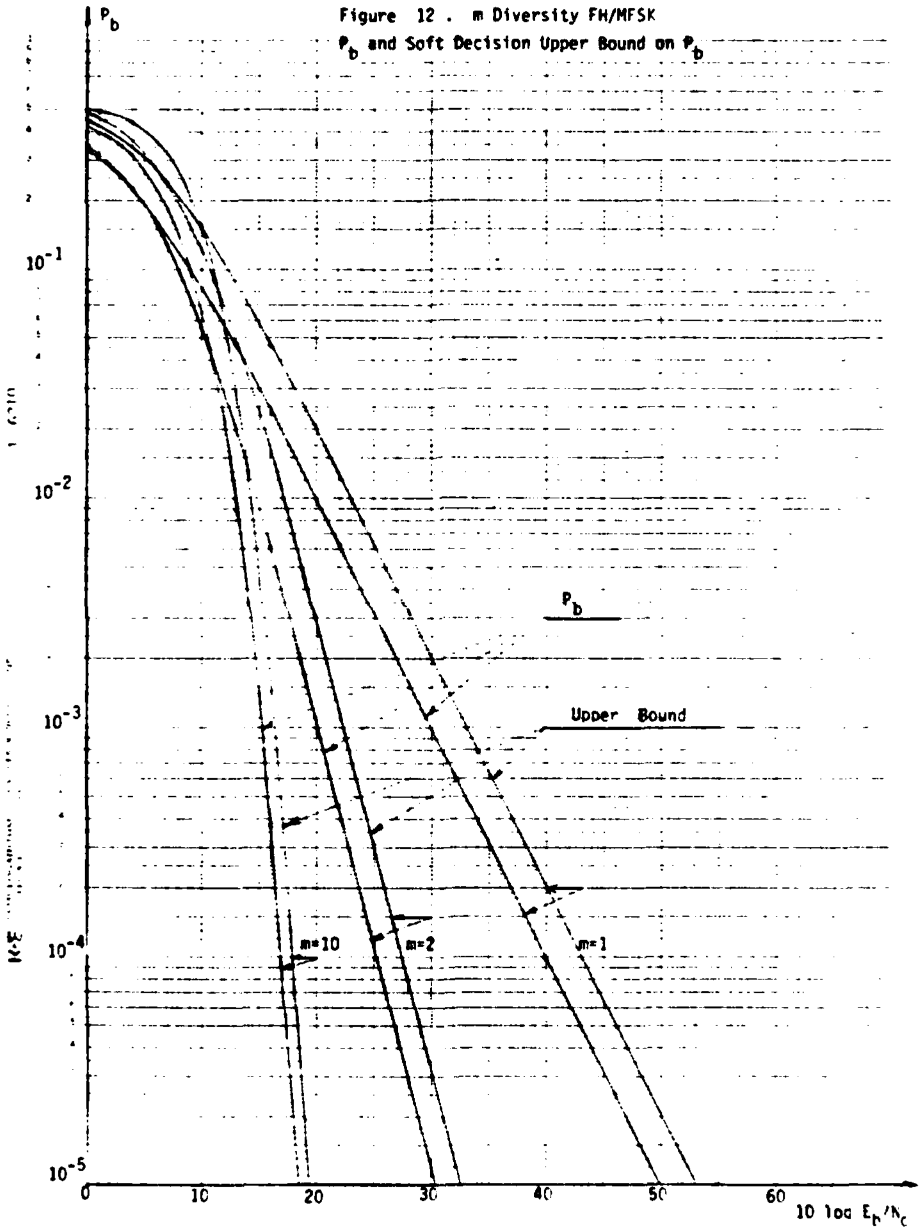


Figure 12 . m Diversity FH/MFSK
P_b and Soft Decision Upper Bound on P_b
P_b
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
10
20
30
40
50
60
10 log E_b/N_c
P_b
Upper Bound
m=10
m=2
m=1

$$P_b \leqslant 2^{K-2} \frac{4(1+K\bar{E}_b/N_o)}{(2+K\bar{E}_b/N_o)^2}$$

$$= 2^K \frac{(1+K\bar{E}_b/N_o)}{(2+K\bar{E}_b/N_o)^2} \tag{6.11}$$

whereas the exact bit error probability is

$$P_b = \frac{M/2}{M-1} P_S$$

$$= \frac{2^{K-1}}{2^k-1} \sum_{k=1}^{2^K-1} \binom{2^K-1}{k} (-1)^{k+1} \frac{1}{1+k(1+K\bar{E}_b/N_o)} \tag{6.12}$$

Both curves are shown in figure 12 for $K=1$.

6.2 m Diversity MFSK

For m diversity MFSK we have the symbol error bound :

$$P_S \leqslant \frac{1}{2}(M-1)\, D\left(\frac{\bar{E}_c}{N_o}\right)^m \tag{6.13}$$

where

$$\bar{E}_c = \frac{K}{m}\bar{E}_b \tag{6.14}$$

Hence

$$P_b = \frac{M/2}{M-1} P_s$$

$$\leqslant \frac{M}{4} D\left(\frac{\bar{E}_c}{N_o}\right)^m \tag{6.15}$$

$$= \frac{M}{4} D\left(\frac{K\bar{E}_b}{mN_o}\right)^m \tag{6.16}$$

For the Soft Decision receiver we then obtain

$$P_b \leq \frac{M}{4}\left\{\frac{4\left(1+\frac{KE_b}{mN_o}\right)}{\left(2+\frac{KE_b}{mN_o}\right)^2}\right\}^m \tag{6.17}$$

We can compare this result with the exact bit error probability obtained by a ML receiver, which for M=2 is known to be [4] :

$$P_b = P^m \sum_{j=0}^{m-1} \binom{m+j-1}{j} (1-P)^j \tag{6.18}$$

where

$$P = \frac{1}{2 + \frac{E_b}{mN_o}} \tag{6.19}$$

Figure 12 shows both curves for several values of m. Given E_b/N_o we can find the optimal value of m and the resultant P_b bound from equation 6.17. Figure 13 shows the P_b bound as a function of E_b/N_o using optimal diversity for M=2,4,8,16,32. The figure shows also the value of the optimal m used to derive each calculated point. Since m can only assume integer values, the smooth curves shown are only approximations to the actual results. As well known [4], for M=2 the optimal diversity is given by

$$m_{opt} \simeq \frac{1}{3} \frac{E_b}{N_o}$$

For the same value of E_b/N_o but higher M, m_{opt} is also higher.

Even when moderate values of signal-to-noise-ratio (say E_b/N_o = 20 dB) are expected, the optimum value of m may well be unrealistically high. A variety of "practical" reasons may preclude

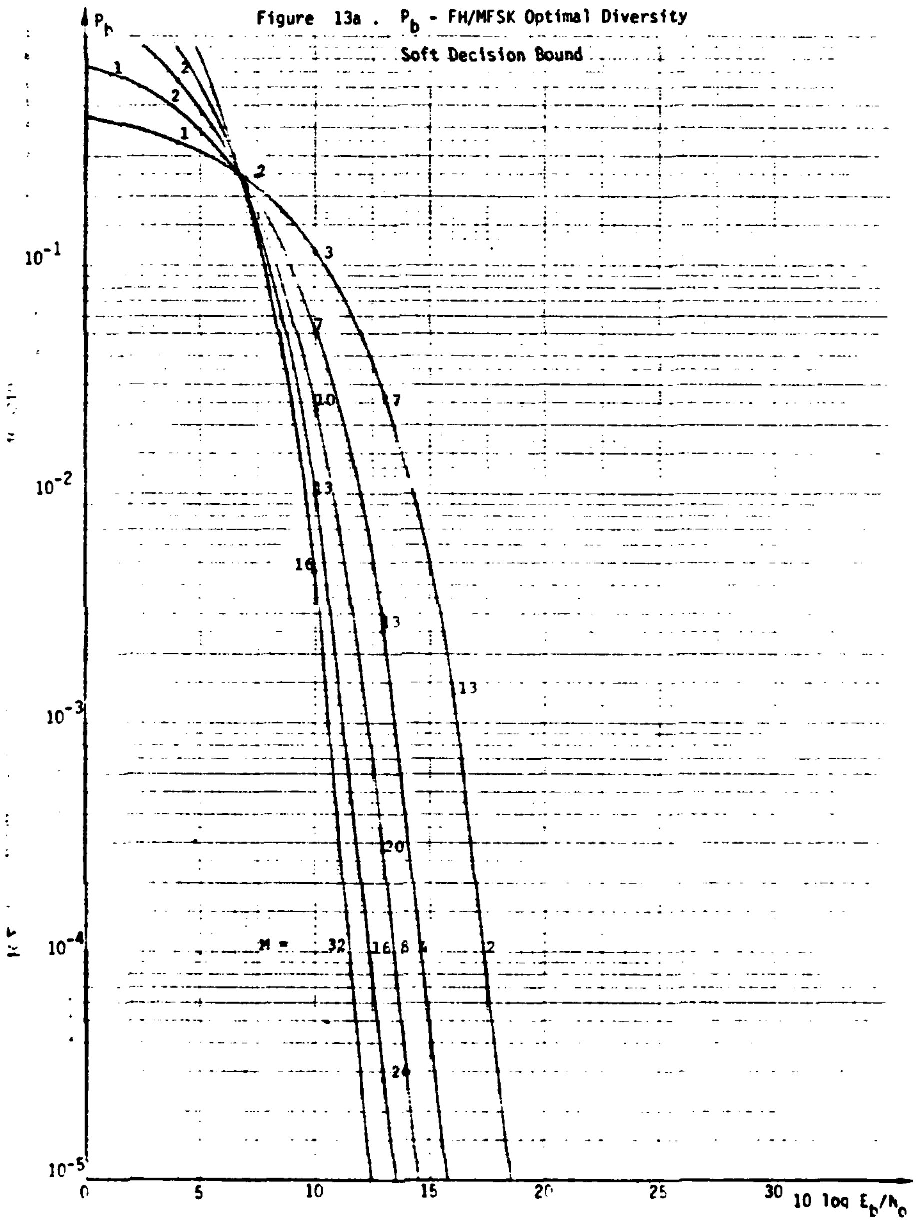
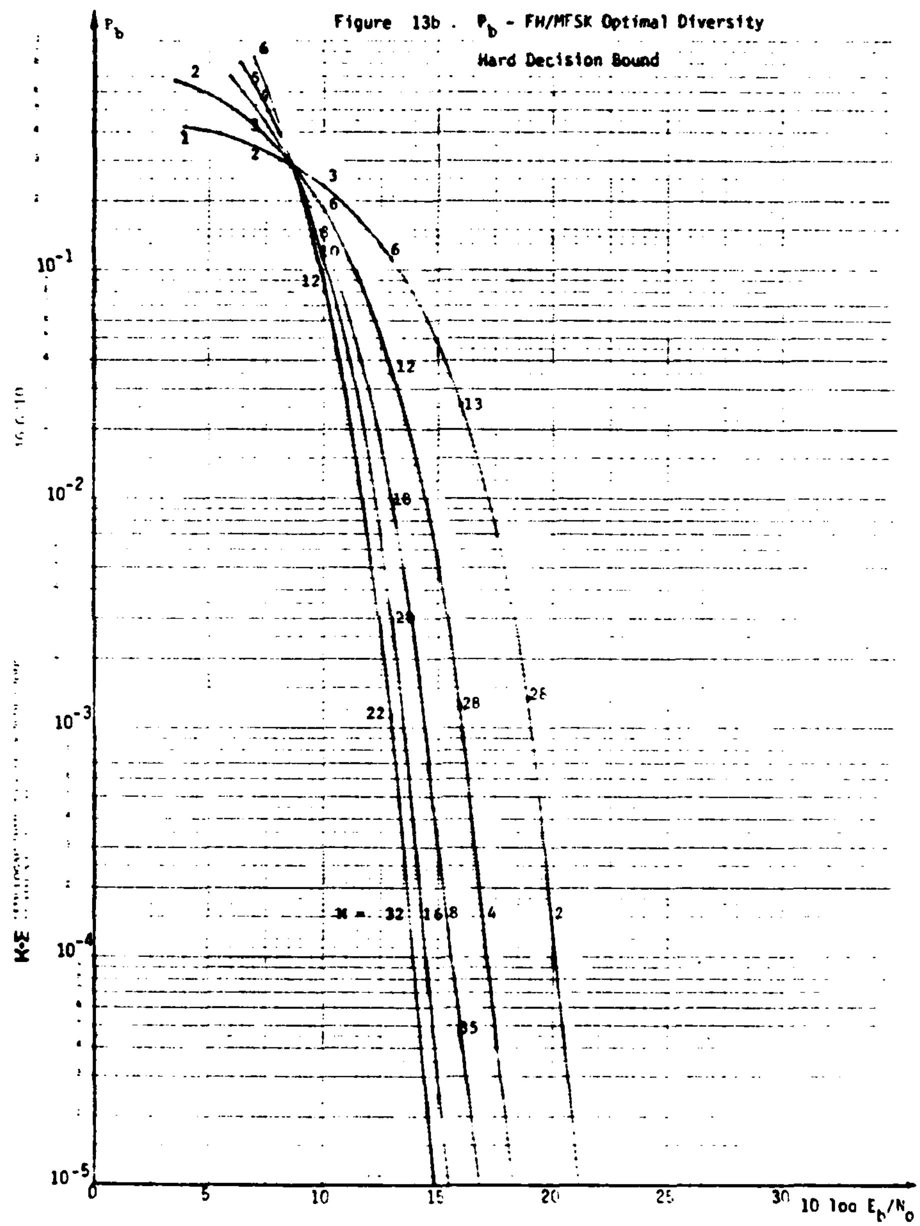


Figure 13a . P_b - FH/MFSK Optimal Diversity
Soft Decision Bound
P_b
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
M = 32 16 8 4 2
0
5
10
15
20
25
30
10 log E_b/N_o



P_b
Figure 13b . P_b - FH/MFSK Optimal Diversity
Hard Decision Bound
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
M = 32 16 8 4 2
0
5
10
15
20
25
30
10 log E_b/N_o

the use of high m values. If, for instance, the information rate is such that the "instantaneous" bandwidth of the transmitted chip is nearly equal to the "coherence bandwidth" of the HF channel, a substantial degradation in performance, not accounted for in our analysis, may appear, when the bit time interval is chopped to shorter chips. Moreover, changing the chip rate so as to follow changes in E_b/N_o is usually undesirable in practice. In such cases it seems reasonable to choose low values of m, so that optimal performance will be achieved when the signal is weak. Figure 13d may be of interest in such a situation. In this Figure we compare the performance of systems using M=2,4,8,16,32, for m=K, i.e., the chip time T_c is equal to T_b for all curves. It can be seen that under such a constraint, high M systems have a profound advantage.

6.3 Orthogonal Convolutional Codes

Conventional MFSK with m diversity is merely a block code containing M code words of blocklength m. We can consider more general codes using M-ary alphabets. An orthogonal convolutional code, for instance, generates one 2^K=M-ary symbols per bit. When used with m diversity, each symbol is "chopped" into m chips and the bit error bound is [3]:

$$P_b \leq \frac{D\left(\frac{E_b}{mN_o}\right)^{mK}}{2\left[1-2D\left(\frac{E_b}{mN_o}\right)^{m}\right]^2} \tag{6.20}$$

6.4 Example

Consider a Soft Decision receiver with JSI and a uniform channel. The information rates are :

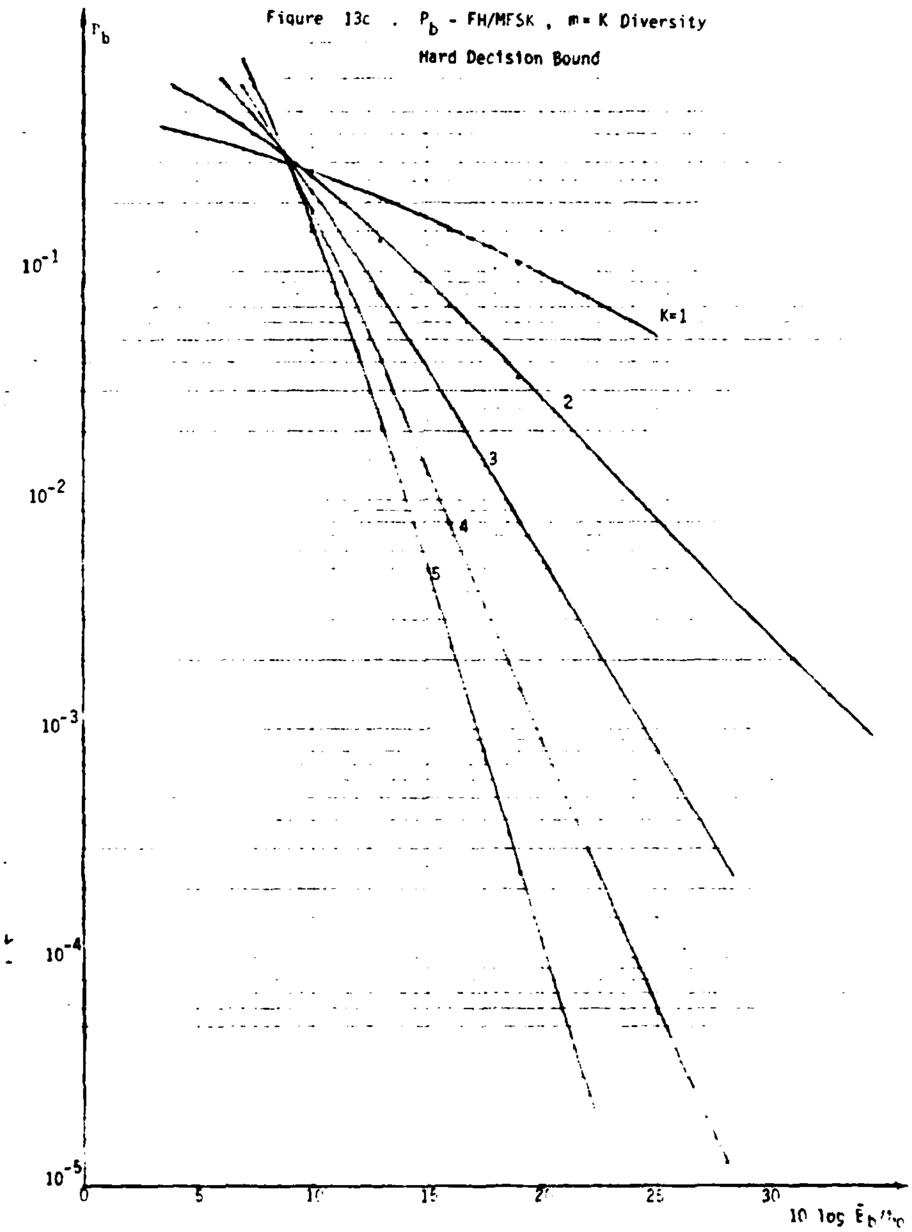


Figure 13c . P_b - FH/MFSK , m = K Diversity
Hard Decision Bound
P_b
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
K=1
2
3
4
5
0
5
10
15
20
25
30
10 log $\bar{E}_b/N_0$

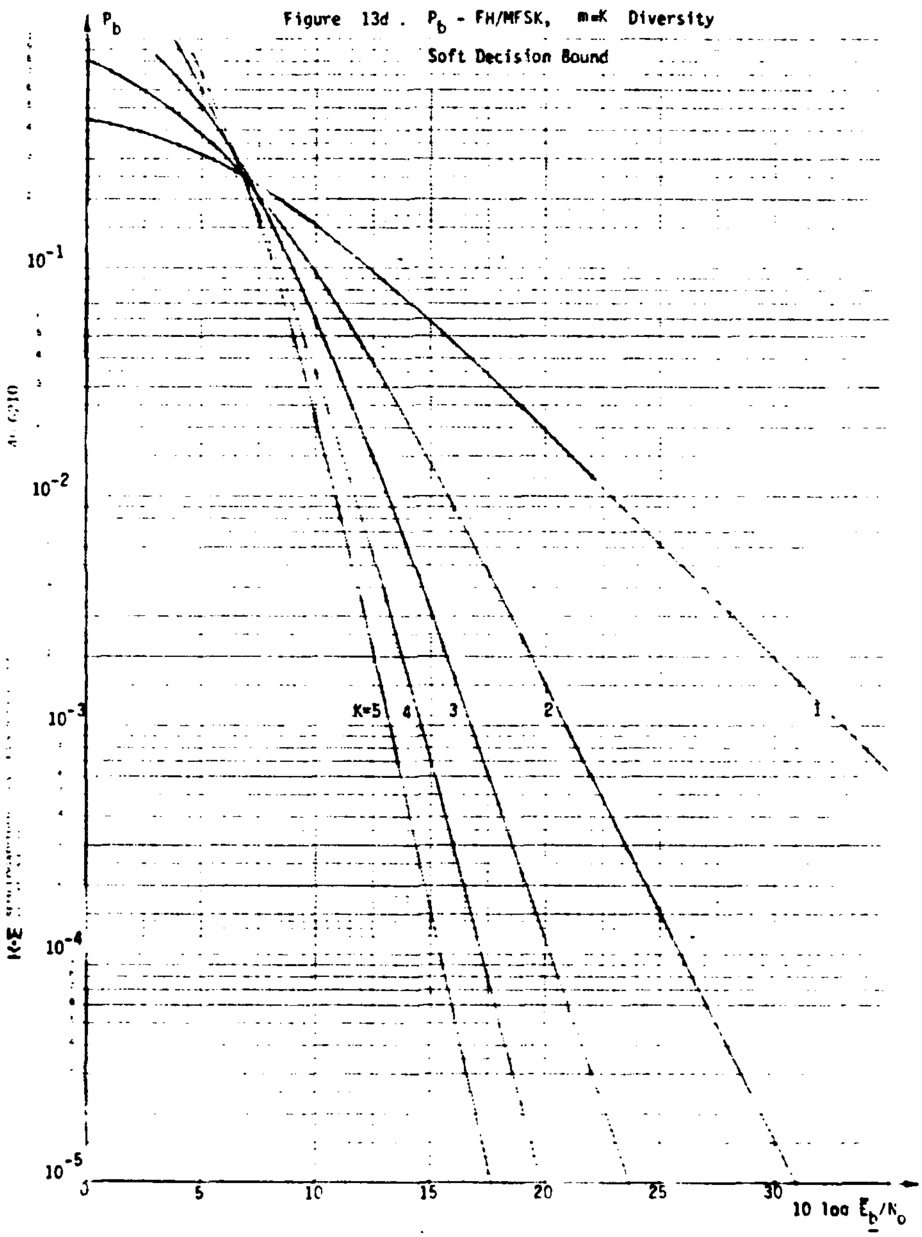
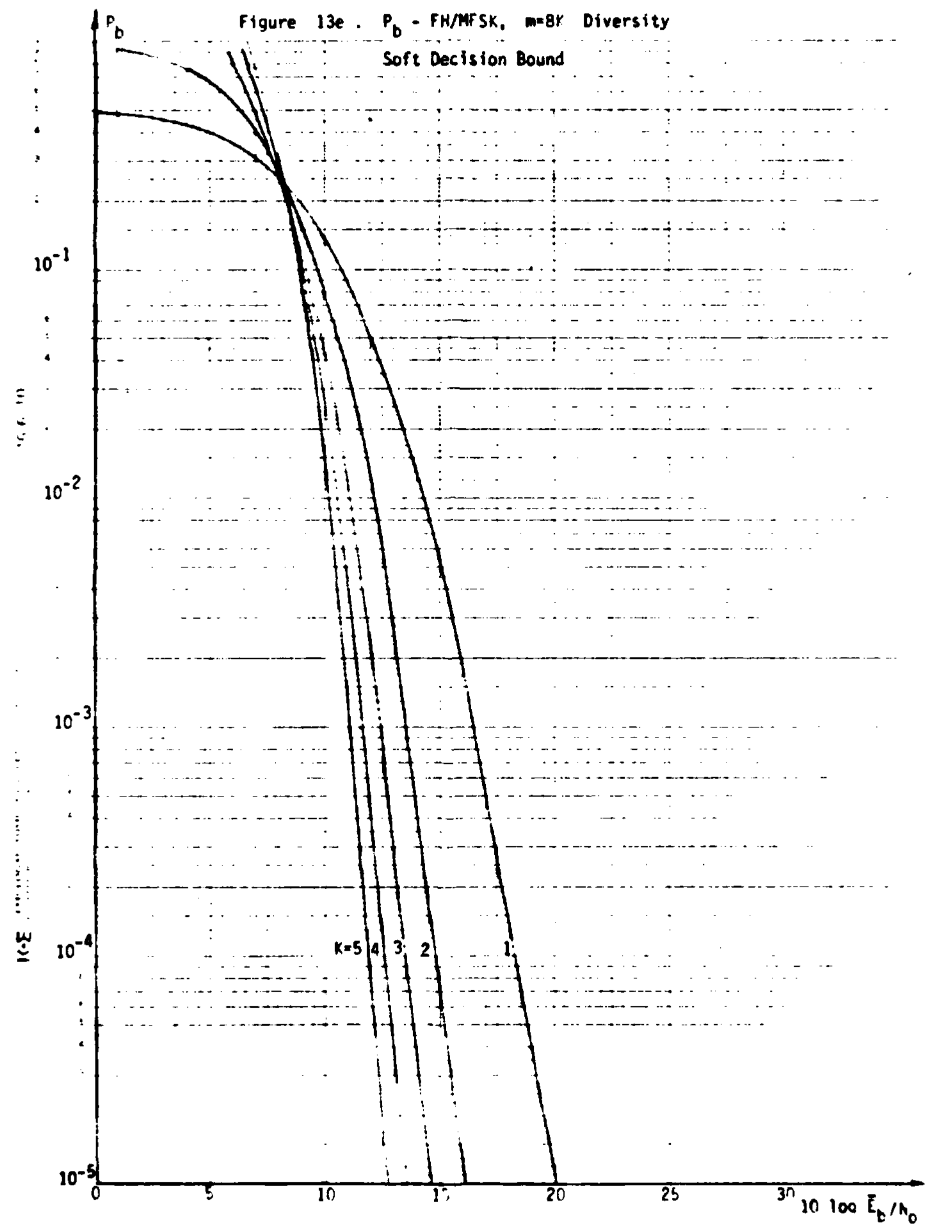


Figure 13d . P_b - FH/MFSK, m=K Diversity
Soft Decision Bound
P_b
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
K=5
4
3
2
1
0
5
10
15
20
25
30
10 log E_b/N_o


P_b
Figure 13e. P_b - FH/MFSK, m=8K Diversity
Soft Decision Bound
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
K=5 4 3 2 1
0
5
10
15
20
25
30
10 log $\bar{E}_b/N_o$

a. R_H = 2400 bits/sec.

b. R_L = 75 bits/sec.

and suppose that at the higher information rate E_b/N_o = 16 dB.

a. For a binary receiver with no diversity, i.e., M=2, K=m=1, we obtain from equations 6.2 and 6.6 :

$$D\left(\frac{E_b}{N_o}\right) = \frac{4(1+39.8)}{(2+39.8)^2} = 0.934\ 10^{-1}$$

and

$$P_b \leqslant \frac{1}{2} D = 0.467\ 10^{-1}$$

The convolutional code from 6.3 yields in this case:

$$P_b \leqslant 0.706\ 10^{-1}$$

Recalling that diversity may help, we see in figure 13 that for this binary receiver at E_b/N_o = 16 dB optimum diversity is m=13. Using this value we obtain:

$$P_b \leqslant \frac{M}{4} D^m$$

$$= \frac{2}{4}\left[\frac{4(1+39.8/13)}{(2+39.8/13)^2}\right]^{13} = 0.134\ 10^{-2}$$

The chip rate is then :

$$R_c = R_H m = 2400 \times 13 = 31{,}200 \text{ chips/second.}$$

If 2400 chips/sec. is the highest permissible chip rate, we can try to use a higher M. For $M=2^K=8$ and m=K=3, the chip rate remains R_c=2400 chips/sec. and $E_c=E_b$.

Hence:

$$P_b \leqslant \frac{M}{4} D\left(\frac{E_b}{N_o}\right)^m = \frac{8}{4}\, 0.0934^3 = 0.162\ 10^{-2}$$

Which is almost as good as optimal diversity for M=2.

The orthogonal convolutional code with M=8, m=3 yields in this case:

$$D\left(\frac{\bar{E}_b}{mN_o}\right) = \frac{4(1+39.8/3)}{(2+39.8/3)^2} = 0.2448$$

$$\therefore \quad P_b \leqslant 0.168\ 10^{-5}$$

b. For the low data rate, $R_L = 75$ bits/sec. :

$$\left(\frac{\bar{E}_b}{N_o}\right)_L = \frac{2400}{75}\left(\frac{\bar{E}_b}{N_o}\right)_H = 32 \times 38.9 = 1.244\ 10^3$$

Hence, for the binary receiver with no diversity we obtain from equations 6.2 & 6.10

$$D = 3.208\ 10^{-3}$$

and

$$P_b \leqslant 0.1604\ 10^{-3}$$

There is no need to use high m in this case. Suppose we take m=4. Then:

$$P_b \leqslant 0.1317\ 10^{-8}$$

and the chip rate:

$$R_c = mR_L = 4 \times 75 = 300 \text{ chips/sec.}$$

CHAPTER VII

OPTIMAL HOPPING STRATEGY OVER A NONUNIFORM CHANNEL

7.1 Optimal Hopping Strategy For an Uncoded Communication System

We now derive the optimum hopping strategy for an uncoded CS threatened by a noise jammer. We assume a "Slotted Channel" as shown in figure 1. The noise density N_j is assumed equal for all sub-channels: $N_j = N_o$; $j=1,\ldots,L$ .The mean received energy per chip of the MFSK signal is defined by the vector :

$$\underline{E} = [E_1, E_2, \ldots, E_L]$$

where

$$E_j = \sigma_j^2 T_b \quad ; \quad j=1,\ldots,L \tag{7.1}$$

Here T_b is the chip duration and σ_j^2 is the mean received power at the j^{th} sub-channel. The jammer divides his power among the L sub-channels according to :

$$\underline{J} = [J_1, J_2, \ldots, J_L]$$

where J_j is the jammer power allocated to the j^{th} sub-channel.

And

$$\sum_{j=1}^{L} J_j = J$$

The contribution of the jammer to the noise power density at the j^{th} sub-band, denoted N_{Jj}, is

$$N_{Jj} = J_j c_j \tag{7.2}$$

We also assume that:

$$E_1 > E_2 > \ldots\ldots > E_L$$

To combat jamming the CS operator is free to "hop" among the sub-channels. When hopping P(j) is the probability that the j^{th} sub-

channel will be used (or the fraction of time sub-channel j is used). The jammer observes $\underline{P}$, but don't know the "hopping" plan" -a random sequence which is adjusted for the desired $\underline{P}$. Based on the information available to him, the jammer chooses $\underline{J}$, subject to:

$$\sum_{j=1}^{L} J_j = J$$

so as to maximize the probability of error of the CS.

For an MFSK noncoherent receiver over a Rayleigh fading channel, the probability of error is given by (See App.I)

$$\varepsilon_j = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1 + k\left(1 + \frac{\bar{E}_j}{N_o + N_{Jj}}\right)} ; \; j=1,\ldots,L \tag{7.3}$$

Suppose now that the jammer is very weak, so that even if the jammer uses all it's power to jam subchannel 1, it would still have lower probability of error than sub-channel 2,...,L. Clearly, in this case the CS operator chooses:

$$\underline{P} = [1,0,0,\ldots,0].$$

i.e., no hopping. Only sub-channel 1 is used.

Now let J_{t1} be defined by :

$$\frac{\bar{E}_1}{N_o + c_1 J_{t1}} = \frac{\bar{E}_2}{N_o}$$

Or :

$$J_{t1} \triangleq \frac{N_o}{c_1} \left[\frac{\bar{E}_1}{\bar{E}_2} - 1 \right] \tag{7.4}$$

Then, J_{t1} is the jamming power that, when used to jam sub-channel 1, makes it exactly as bad as sub-channel 2 (unjammed).

It is intuitively clear that for $J > J_{t1}$, the CS operator should hop

between sub-channel 1 and other sub-channels.

We now give a formal definition of the problem : Let

$$\underline{P}^* = [P^*(1), P^*(2), \ldots, P^*(L)]$$

and

$$\underline{J}^* = [J_1^*, J_2^*, \ldots, J_L^*]$$

be the minimax solution, i.e.:

$\underline{J}^*$ maximizes $P_b(\underline{P}^*, \underline{J})$ over all possible $\underline{J}$ subject to :

$$\sum_{j=1}^{L} J_j = J$$

where

$$P_b(\underline{P}^*, \underline{J}) = \sum_{j=1}^{L} P^*(j)\epsilon_j$$

$$= \sum_{j=1}^{L} P^*(j) \sum_{K=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left[1+\frac{E_j}{N_o+c_j J_j}\right]} \tag{7.5}$$

and $\underline{P}^*$ is the probability vector which minimizes the maximum $P_b(\underline{P}, \underline{J})$ that the jammer can achieve knowing $\underline{P}$.

Or, formally

$$\underline{J}^* = \max_{\underline{J}}{}^{-1} P_b(\underline{P}^*, \underline{J}) \tag{7.6}$$

and

$$\underline{P}^* = \min_{\underline{P}}{}^{-1} \max_{\underline{J}} P_b(\underline{P}, \underline{J}) \tag{7.7}$$

We want to find $\underline{P}^*$ and $\underline{J}^*$.

Using the Lagrange multiplier λ_1 for the constraint

$$\sum_{j=1}^{L} P(j) = 1 ,$$

Let :

$$F_1(\underline{P},\underline{J},\lambda_1) = \sum_{j=1}^{L} P(j) \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+\frac{E_J}{N_o+c_jJ_j}\right)} + \lambda_1\left[\sum_{j=1}^{L} P(j)-1\right]$$

Then, the minimax point $(\underline{P}^*,\underline{J}^*)$ must satisfy the conditions ([3], Appendix 3B.1 - KUHN - TUCKER conditions) :

$$\left.\begin{aligned} &\left.\frac{\partial F_1(\underline{P},\underline{J}^*,\lambda_1)}{\partial P(j)}\right|_{P(j)=P^*(j)} = 0 \quad ; \text{all } j \text{ such that } P^*(j)>0 \\ &\sum_{j=1}^{L} P(j) = 1 \end{aligned}\right\} \mathrm{I} \tag{7.7}$$

Also let

$$F_2(\underline{P},\underline{J},\lambda_2) = \sum_{j=1}^{L} P(j) \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+\frac{\bar{E}_j}{N_o+c_jJ_j}\right)} + \lambda_2\left[\sum_{j=1}^{L} J_j - J\right] \tag{7.8}$$

Then, the minimax point should also satisfy the conditions:

$$\left.\begin{aligned} &\left.\frac{\partial F_2(\underline{P}^*,\underline{J},\lambda_2)}{\partial J_j}\right|_{J_j=J_j^*} = 0 \quad ; \text{all } j \text{ such that } P^*(j)>0 \\ &J_j^* = 0 \quad ; \text{all } j \text{ such that } P^*(j)=0 \\ &\sum_{j=1}^{L} J_j^* = J \end{aligned}\right\} \mathrm{II} \tag{7.9}$$

From condition I we obtain :

$$\sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k\left(1+\frac{\bar{E}_j}{N_o+c_jJ_j^*}\right)} + \lambda_1 = 0 \quad ; \text{all } j \text{ such that } P^*(j)>0$$

This implies that

$$\frac{E_j}{N_o+c_jJ_j^*} = \epsilon \quad ; \text{all } j \text{ such that } P^*(j)>0$$

This implies that at the minimax solution the probability of error P_{bj} of all the active sub-channels must be equal. It also implies that the number of active sub-channels depends on the jammer power J. It was mentioned above that for $J>J_{t1}$, more than a single sub-channel is used, while for $J\leq J_{t1}$ only sub-channel 1 is used. Similarly we define J_{tn}, $n<L$ to be the highest jammer power for which only n sub-channels are used for hopping. Also, let J_{tn}^i be the jammer power allocated to sub-channel i when $J=J_{tn}$.

Hence:

$$J_{tn}=\sum_{i=1}^{n} J_{tn}^{i}$$

Where

$$\frac{E_1}{N_o+c_1J_{tn}^1}=\frac{E_2}{N_o+c_2J_{tn}^2}=\dots\dots=\frac{E_n}{N_o+c_nJ_{tn}^n}=\frac{E_{n+1}}{N_o}$$

Then :

$$J_{tn}^{i}=\frac{N_o}{c_i}\left[\frac{E_i}{E_{n+1}}-1\right] \quad ; \quad i=1,2,\dots,n$$

Hence :

$$J_{tn}=\begin{cases}\dfrac{N_o}{E_{n+1}}\sum\limits_{i=1}^{n}\dfrac{E_i}{c_i}-N_o\sum\limits_{i=1}^{n}\dfrac{1}{c_i} & ; \quad n<L\\ \infty & ; \quad n\nless L\end{cases} \tag{7.10}$$

Therefore, knowing the jammer power, the number N of sub-channels, the CS should use for Hopping, is to be found from:

$$J_{t,N-1}<J\leq J_{tN} \tag{7.11}$$

We have seen that the minimax solution corresponds to :

$$\frac{\bar{E}_j}{N_o + c_j J_j^*} = \epsilon \qquad : \; j=1,\ldots,N$$

$$\therefore \; J_j^* = \frac{\bar{E}_j}{\epsilon c_j} - \frac{N_o}{c_j} \qquad ; \; j=1,\ldots,N$$

$$\therefore \; J = \frac{1}{\epsilon} \sum_{j=1}^{N} \frac{\bar{E}_j}{c_j} - N_o \sum_{j=1}^{N} \frac{1}{c_j}$$

Or :

$$\epsilon = \frac{\sum_{j=1}^{N} \frac{\bar{E}_j}{c_j}}{J + N_o \sum_{j=1}^{N} \frac{1}{c_j}}$$

Hence:

$$J_j^* = \begin{cases} \bar{E}_j \dfrac{J + N_o \sum_{i=1}^{N} \frac{1}{c_i}}{c_j \sum_{i=1}^{N} \frac{\bar{E}_i}{c_i}} - \dfrac{N_o}{c_j} & ; \; j=1,\ldots,N \\ 0 & ; \; j>N \end{cases} \tag{7.12}$$

To find $\underline{P}^*$ we use condition II :

Let

$$x_j(J_j) \triangleq 1 + \frac{E_j}{N_o + c_j J_j} \quad ; \quad j=1,\ldots,N$$

Hence:

$$x_j(J_j^*) \triangleq 1 + \frac{E_j}{N_o + c_j J_j^*} = 1 + \epsilon$$

Then

$$\frac{\partial F_2(\underline{P}^*, J, \lambda_2)}{\partial J_j} = P^*(j) \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{(-k)}{(1+kx_j)^2} \frac{dx_j}{dJ_j} + \lambda_2$$

$$j=1,2,\ldots,N$$

Where

$$\frac{dx_j}{dJ_j} = - \frac{c_j E_j}{(N_o + c_j J_j)^2}$$

Hence, condition II implies:

$$\frac{P^*(j) c_j}{E_j} \epsilon^2 \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{k}{[1 + k(1+\epsilon)]^2} + \lambda_2 = 0 \; ; \; j=1,\ldots,N$$

Or

$$\frac{P^*(j) c_j}{E_j} = C \; ; \qquad j=1,\ldots,N$$

But

$$\sum_{j=1}^{N} P^*(j) = 1$$

Hence

$$C \sum_{j=1}^{N} \frac{E_j}{c_j} = 1$$

$$P^*(j) = \begin{cases} \dfrac{E_j/c_j}{\sum_{j=1}^{N} \frac{E_j}{c_j}} & ; \; j=1,\ldots,N \\ 0 & ; \; j>N \end{cases} \tag{7.13}$$

7.2 Extension to Coded Systems

Our aim now is to show that the solution to the original minimax problem, which delt with the probability of error of a channel used with no coding, remains a valid minimax solution for $D(\underline{P})$, which is the Hard Decision bound parameter:

$$D(\underline{P}) = \sum_{j=1}^{N} P(j)\left[2\sqrt{\frac{(1-\varepsilon_j)\varepsilon_j}{M-1}} + \frac{M-2}{M-1}\varepsilon_j\right]$$

Where ε_j is the bit error probability of the j^{th} sub-channel. Recall from 7.1 that if $\underline{P}^*$ is the minimax solution for P_b, where :

$$P_b = \sum_{j=1}^{N} P(j)\varepsilon_j(J_j)$$

and $\underline{\varepsilon}^* = (\varepsilon_1^*, \varepsilon_2^*, \ldots, \varepsilon_N^*)$ is the associated probability of error of the channel, which correspond to the worst case jamming power distribution:

$$\underline{J}^* = (J_1^*, J_2^*, \ldots, J_N^*)$$

then:

$$\underline{P}^* = [P^*(1), P^*(2), \ldots, P^*(N)]$$

is the solution of the following system of $N + 1$ equations:

$$P(j)\frac{d(\varepsilon_j)}{d_{Jj}} + \lambda\bigg|_{Jj = J_j^*} = 0 \quad ; \quad j=1,\ldots,N$$

$$\sum_{j=1}^{N} P(j) = 1$$

and $\underline{J}^*$ is chosen such that :

$$\varepsilon_1(J_1^*) = \varepsilon_2(J_2^*) = \ldots\ldots = \varepsilon_n(J_n^*)$$

$$\sum_{j=1}^{N} J_j^* = J$$

Similarly, since

$$D(\underline{P},\underline{J}) = \sum_{j=1}^{N} P(j)\left[2\sqrt{\frac{\varepsilon_j(1-\varepsilon_j)}{M-1}} + \frac{M-2}{M-1}\varepsilon_j\right]$$

is a convex $\cap$ function of $\underline{J}$, the minimax solution $\underline{\overline{P}}$ for $D(\underline{P},\underline{J})$ is to be found from:

$$\overline{P}(j)\frac{d}{dJ_j}\left[2\sqrt{\frac{\varepsilon_j(1-\varepsilon_j)}{M-1}} + \frac{M-2}{M-1}\varepsilon_j\right] + \lambda \Bigg|_{J_j = J_j^*} = 0 \; ; j=1,\ldots,N$$

$$\sum_{j=1}^{N} \overline{P}(j) = 1$$

where $\underline{J}^*$ is the unique vector such that :

$$\varepsilon_1(J_1^*) = \varepsilon_2(J_2^*) = \ldots = \varepsilon_N(J_N^*) \triangleq \varepsilon$$

and

$$\sum_{j=1}^{N} J_j^* = J$$

but

$$\frac{d}{dJ_j}\left[2\sqrt{\frac{\varepsilon_j(1-\varepsilon_j)}{M-1}} + \frac{M-2}{M-1}\varepsilon_j\right]\Bigg|_{J_j = J_j^*}$$

$$= \frac{d}{d\varepsilon_j}\left[2\sqrt{\frac{\varepsilon_j(1-\varepsilon_j)}{M-1}} + \frac{M-2}{M-1}\varepsilon_j\right]\left(\frac{d\varepsilon_j}{dJ_j}\right)\Bigg|_{J_j = J_j^*}$$

$$= \left|\frac{d}{d\varepsilon_j}\left[2\sqrt{\frac{\varepsilon_j(1-\varepsilon_j)}{M-1}} + \frac{M-2}{M-1}\varepsilon_j\right]\right|_{\varepsilon_j = \varepsilon} \cdot \left|\left(\frac{d\varepsilon_j}{dJ_j}\right)\right|_{J_j = J_j^*}$$

$$= Q(\varepsilon)\left|\frac{d\varepsilon_j}{dJ_j}\right|_{J_j = J_j^*}$$

where

$$Q(\varepsilon) \triangleq \left|\frac{d}{d\varepsilon_j}\left[2\sqrt{\frac{\varepsilon_j(1-\varepsilon_j)}{M-1}} + \frac{M-2}{M-1}\varepsilon_j\right]\right|_{\varepsilon_j = \varepsilon}$$

Hence, the equations that we must solve have the form:

$$Q(\varepsilon)\bar{P}(j)\left|\frac{d\varepsilon_j}{dJ_j}\right|_{J_j = J_j^*} + \lambda = 0 \quad ; \quad j=1,\ldots,N$$

$$\sum_{j=1}^{N} \bar{P}(j) = 1$$

or:

$$\bar{P}(j)\left|\frac{d\varepsilon_j}{dJ_j}\right|_{J_j = J_j^*} + \frac{\lambda}{Q(\varepsilon)} = 0 \quad ; \quad j=1,\ldots N$$

$$\sum_{j=1}^{N} \bar{P}(j) = 1 \tag{7.14}$$

Comparing equation 7.14 to equation 7.9 , it is clear that both have the same solution, i.e., $\bar{\underline{P}}(j) = \underline{P}(j)$ (and only the value of λ is different - which is of no consequence).

It can be similarly shown that the minimax solution, found above, applies also to the Soft Decision bound:

$$D(\underline{P},\underline{J}) = \sum_{j=1}^{L} P(j) \frac{4\left(1 + \frac{E_j}{N_j+C_jJ_j}\right)}{\left(2 + \frac{E_j}{N_j+C_jJ_j}\right)^2}$$

CHAPTER VIII

MULTI TONE JAMMING

8.1 Introductory Discussion and Definitions

In this chapter we consider FH / BFSK signalling in Rayleigh fading channels which are subject to multi-tone jamming. This kind of jamming differs from noise jamming in several important characteristics some of which we discuss briefly below.

It is intuitively clear that a multi-tone jammer is most effective when it hits no more than one tone in each M tone sub-band. In a way, hitting two tones in the same sub-band constitutes a waste of jamming power. Consequently, given the total spread spectrum bandwidth W, the available number of M tone sub-bands, and the total number of tones the jammer must use in order to "cover" a certain fraction of these sub-bands, is inversely proportional to M . Hence, the power of each tone the jammer transmits is proportional to M, i.e. larger M implies increased jammer effectivity. Therefore, FH / BFSK yields better performance under multi-tone jamming than FH / MFSK signalling for $M > 2$. In view of this fact the following analysis deals with the binary case only.

In the noise jamming case we assumed that under favorable conditions the receiver may be able to detect the presence or absence of the jammer's emission in the currently used sub-band for each chip time interval. Under multi-tone jamming, however, both the friendly

transmitter and the jammer may hit the same tone position, and therefore, the receiver cannot always (or nearly so) detect the presence of the jammer. The concept of JSI is therefore unrealistic when multi - tone jammer is considered. In spite of the above, when N- the number of active sub-bands, is moderately large, the receiver may still be able to measure ρ and therefore, a receiver having knowledge of ρ is included in the following study.

Recall that in the noise jamming case we have used the random variable Z_n , $n=1,\ldots,m$, to specify whether the n^{th} chip signal was jammed or not. In the following, Z_n is still used in the same manner, but in addition we introduce the binary random variable j_n defined as follows:

$$j_n = \begin{cases} 0\,, & \text{the jammer hits } \omega_0 \text{ during the } n^{th} \text{ chip time} \\ 1\,, & \text{the jammer hits } \omega_1 \text{ during the } n^{th} \text{ chip time} \end{cases} \qquad n=1,\ldots,m$$

we also assume that:

$$P_{j_n}(0) = P_{j_n}(1) = 1/2 \quad ; \quad n=1,\ldots,m$$

and that the receiver knows this fact.

In this work we assume that both, the signal and the multi - tone jammer are subject to Rayleigh fading. Assuming that the $\ell_n{}^{th}$ sub-band was used during the n^{th} chip time, the received jammer signal (following the dehopper) is of the form:

$$B_{\ell_n}\cos(\omega_i t + \theta_n) \quad , \quad i \in [0,1] \quad , \quad n=1,\ldots,m$$

where:

$$p_{B_{\ell_n}}(b) = \frac{b}{\sigma^2_{J\ell_n}} \exp\left\{-\frac{b^2}{2\sigma^2_{J\ell_n}}\right\} \qquad ; \; b>0$$

and

$$P_{\theta_n}(\theta) = \begin{cases} \frac{1}{2\pi} & , \quad 0 \leq \theta < 2\pi \\ 0 & , \quad \text{elsewhere} \end{cases} \qquad n=1,\ldots,m$$

This implies that $E\left[B^2_{\ell_n}\right] = 2\sigma^2_{J\ell_n}$

where σ^2_{Jj} - the mean received jammer power at the j^{th} sub-band is given by:

$$\sigma^2_{Jj} = J_j a_j \tag{8.1}$$

For convenience we also define:

$$E_{Jj} = \sigma^2_{Jj} T_c$$

$$= J_j c_j \tag{8.2}$$

We now turn our attention to specific receivers.

8.2 Performance Analysis Of Specific Receivers

8.2.1 Hard decision FH /BFSK receiver under pulsed/partial band multi-tone jamming.

We begin with the uniform channel. The input and output alphabets are:

$$X = Y \in [0,1]$$

The conditional probability of $\underline{y}$ given $\underline{x}$ is:

$$P_m(\underline{y}/\underline{x}) = \prod_{n=1}^{m} P(y_n/x_n)$$

But when no jammer is present:

$$P(y_n/x_n, Z_n=0) = \begin{cases} 1-P_2 & ; \quad y_n = x_n \\ P_2 & ; \quad y_n \neq x_n \end{cases} \tag{8.3}$$

Where:

$$P_2 = \frac{1}{2 + \overline{E}_c/N_o}$$

while under jamming :

$$P(y_n/x_n, Z_n=1, j_n) = \begin{cases} 1-P_1 & ; \quad y_n = x_n = j_n \\ 1-P_o & ; \quad y_n = x_n \neq j_n \\ P_1 & ; \quad y_n \neq x_n = j_n \\ P_o & ; \quad y_n \neq x_n \neq j_n \end{cases} \tag{8.4}$$

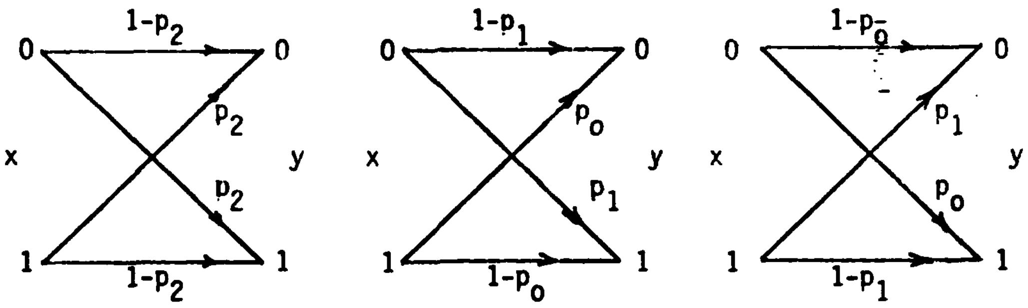
Where :

$$P_1 = \frac{1}{2 + \frac{\overline{E}_c}{N_o} + \frac{\overline{E}_j}{N_o \rho}}$$

$$P_o = \frac{1 + \frac{\overline{E}_j}{N_o \rho}}{2 + \frac{\overline{E}_c}{N_o} + \frac{\overline{E}_j}{N_o \rho}}$$

(See appendix II for the derivation of p_1 & p_o)

Pictorially:



Z_n=0 j=0 j=1

Hence:

$$P(y_n/x_n) = (1-\rho)P(y_n/x_n,Z_n=0) + \rho P(y_n/x_n,Z_n=1)$$

$$= (1-\rho)P(y_n/x_n,Z_n=0)+\rho\Big[P(y_n/x_n,Z_n=1,j_n=0)/2 + P(y_n/x_n,Z_n=1,j_n=1)/2\Big]$$

and:

$$P(y_n/x_n) = \begin{cases} (1-\rho)(1-p_2) + \rho(1-\dfrac{p_o+p_1}{2}) & ; \quad y=x \\ (1-\rho)p_2 + \rho(\dfrac{p_o+p_1}{2}) & ; \quad y \neq x \end{cases} \tag{8.5}$$

where :

$$\frac{p_o+p_1}{2} = \frac{1}{2}\,\frac{2 + \dfrac{\bar{E}_J}{N_o\rho}}{2 + \dfrac{\bar{E}}{N_o} + \dfrac{\bar{E}_J}{N_o\rho}}$$

- Without JSI the receiver uses the metric :

$$m(y_n;x_n) = -W(y_n;x_n) \tag{8.6}$$

which is a ML metric (can be written in the form :

$$m(y_n;x_n) = a\ell nP(y_n/x_n) + b \quad)$$

Note that the receiver does not need to know ρ in order to use this metric. Hence, the performance is bounded by the Bhattacharyya bound:

$$\min_{o<\lambda} D(x_n,\hat{x}_n;\lambda) \leq \sum_{y_n} \sqrt{P(y_n/\hat{x}_n)P(y_n/x_n)} \quad ; \hat{x}_n \neq x_n$$

$$=2\sqrt{\left[(1-\rho)\frac{1+\frac{\bar{E}_c}{No}}{2+\frac{\bar{E}_c}{N_o}}+\rho\left(1-\frac{1}{2}\frac{2+\bar{E}_J/N_o\rho}{2+\frac{\bar{E}_c}{N_o}+\frac{\bar{E}_J}{N_o\rho}}\right)\right]\left[\frac{1-\rho}{2+\frac{\bar{E}_c}{N_o}}+\frac{\rho}{2}\frac{2+\bar{E}_J/N_o\rho}{2+\frac{\bar{E}_c}{N_o}+\frac{\bar{E}_J}{N_o\rho}}\right]}$$

$$=2\sqrt{\left\{(1-\rho)\frac{1+\bar{E}_c/N_o}{2+\bar{E}_c/N_o}+\rho\frac{1+\frac{\bar{E}_c}{N_o}+\frac{\bar{E}_J}{2N_o\rho}}{\frac{\bar{E}_c}{N_o}+\frac{\bar{E}_J}{N_o\rho}}\right\}\left[\frac{1-\rho}{2+\frac{\bar{E}_c}{N_o}}+\frac{1+\bar{E}_J/2N_o\rho}{2+\frac{\bar{E}_c}{N_o}+\frac{\bar{E}_J}{N_o\rho}}\rho\right]}$$

$$\triangleq D_\rho\left(\frac{\bar{E}_c}{N_o};\frac{\bar{E}_J}{N_o}\right)$$

The value of ρ which maximizes D is again $\rho=1$. Hence:

$$D_{wc} = \max_{o<\rho<1} D = 2\,\frac{\sqrt{\left(1+\frac{\bar{E}_c}{N_o}+\frac{\bar{E}_J}{2N_o}\right)\left(1+\frac{\bar{E}_J}{2N_o}\right)}}{2+\frac{\bar{E}_c}{N_o}+\frac{\bar{E}_J}{N_o}} \tag{8.7}$$

This can be written in a concise form by letting:

$$\varepsilon_M \triangleq \frac{1+\bar{E}_J/2N_o}{2+\frac{\bar{E}_c}{N_o}+\frac{\bar{E}_J}{N_o}}$$

Then:

$$D_{wc} = 2\sqrt{\varepsilon_M(1-\varepsilon_M)}$$

and

$$P(\underline{x} \to \underline{\hat{x}}) \leq D_{wc}^{W(\underline{x};\underline{\hat{x}})}$$

Since ε_M depends only on the ratio E_c/N_o and E_J/N_o, it is convenient to define:

$$\Psi_c \triangleq E_c/N_o$$

and

$$g \triangleq E_J/E_c$$

Then:

$$\varepsilon_M(\Psi_c, g) = \frac{1 + \Psi_c g/2}{2 + \Psi_c(1+g)} \tag{8.8}$$

When $E_c/N_o >> 1$ and $E_J/N_o >> 1$, $\varepsilon_M(\Psi_c, g)$ reduces to:

$$\varepsilon_M \approx \frac{g}{2(1+g)} \tag{8.9}$$

A trivial extension of this derivation yields a similar bound for the Slotted Channel:

$$D_{wc} = \sum_{j=1}^{N} P(j) 2\sqrt{\varepsilon_{Mj}(1-\varepsilon_{Mj})} \tag{8.10}$$

where:

$$\varepsilon_{Mj} = \frac{1 + E_J/N_j 2}{2 + \frac{E_j}{N_j} + \frac{E_{Jj}}{N_j}} \tag{8.11}$$

8.2.2 Soft Decision receiver having knowledge of ρ over a negligible noise Slotted Channel under pulsed / partial band multi-tone

jammer.

The input alphabet is : $x \in \{0,1\}$,

The output alphabet is:

$$\underline{y}^{(n)} = (y_0^{(n)}, y_1^{(n)}) \quad ; \quad n=1,\ldots,m$$

Where $y_k^{(n)}$ is the k^{th} detector output at the n^{th} chip time.

The conditional probability of $\underline{y}$ given $\underline{x}$ and the particular hopping sequence $\underline{L}$ is:

$$P_{2m}(\underline{y}/\underline{x},\underline{L}) = \prod_{n=1}^{m} P_2(\underline{y}^{(n)}/x^{(n)},\ell_n)$$

where:

$$P_2(\underline{y}^{(n)}/x^{(n)},\ell_n) = \sum_{k=o}^{1} P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,k)P_{Z_n}(k)$$

$$=P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,0)P_{Z_n}(0) + P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,1)P_{Z_n}(1)$$

$$=(1-\rho)P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,0) + \rho P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,1)$$

$$=(1-\rho)P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,0) +$$

$$+ \frac{\rho}{2}\left[P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,1,j_n=x^{(n)}) + P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,1,j_n\neq x^{(n)})\right]$$

But:

$$P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,0) = P\left(y_{x^{(n)}}^{(n)}/x^{(n)},\ell_n,0\right)P\left(y_{\bar{x}^{(n)}}^{(n)}/x^{(n)},\ell_n,0\right)$$

$$= \frac{2\,y_{x^{(n)}}^{(n)}}{\bar{E}_{\ell_n}} \exp\left\{-\frac{y_{x^{(n)}}^{(n)}}{\bar{E}_{\ell_n}}\right\}\delta\left(y_{\bar{x}^{(n)}}^{(n)}\right)$$

Where

$$\bar{x} = \begin{cases} 0 & ; \quad x = 1 \\ 1 & ; \quad x = 0 \end{cases}$$

and:

$$P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,1,j_n=x^{(n)}) = P(y^{(n)}_{x^{(n)}}/x^{(n)},\ell_n,1,j_n=x^{(n)}) \cdot$$

$$\cdot\ P(y^{(n)}_{\bar{x}^{(n)}}/x^{(n)},\ell_n,1,j_n=x^{(n)})$$

$$= \frac{2y^{(n)}_{x^{(n)}}}{\bar{E}_{\ell_n} + \bar{E}_{J\ell_n}/\rho} \exp\left\{-\frac{y^{(n)^2}_{x^{(n)}}}{\bar{E}_{\ell_n} + \bar{E}_{J\ell_n}/\rho}\right\}\delta\left(y^{(n)}_{\bar{x}^{(n)}}\right)$$

Whereas:

$$P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,1,j_n=\bar{x}^{(n)}) = P(y^{(n)}_{x^{(n)}}/x^{(n)},\ell_n,1,j_n=\bar{x}^{(n)})\cdot$$

$$\cdot P(y^{(n)}_{\bar{x}^{(n)}}/x^{(n)},\ell_n,1,j_n=\bar{x}^{(n)})$$

$$= \frac{2y^{(n)}_{x^{(n)}}}{\bar{E}_{\ell_n}} \exp\left\{-\frac{y^{(n)^2}_{x^{(n)}}}{\bar{E}_{\ell_n}}\right\} \frac{2y^{(n)}_{\bar{x}^{(n)}}}{\bar{E}_{J\ell_n}/\rho} \exp\left\{-\frac{y^{(n)^2}_{\bar{x}^{(n)}}}{\bar{E}_{J\ell_n}/\rho}\right\}$$

To use the ML metric we take:

$$m(\underline{y}^{(n)};x^{(n)}/\ell_n) = \ell n P_2(\underline{y}^{(n)}/x^{(n)},\ell_n)$$

$$= \ell n\Big\{(1-\rho)P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,0) +$$

$$\frac{\rho}{2}\left[P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,1,j_n=x^{(n)}) + P_2(\underline{y}^{(n)}/x^{(n)},\ell_n,1,j_n=\bar{x}^{(n)})\right]\Big\} \tag{8.12}$$

The Bhattacharyya bound parameter D is then given by:

$$D = E\left\{ \int_{\underline{y}^{(n)}} \sqrt{P_2(\underline{y}^{(n)}/x^{(n)},\ell_n)P_2(\underline{y}^{(n)}/x^{(n)},\ell_n)}\, dy_1^{(n)} dy_2^{(n)} \Big/ \underset{\hat{x}^{(n)} \neq \bar{x}^{(n)}}{x^{(n)}} \right\}$$

$$= \sum_{j=1}^{N} P(j) \int_{\underline{y}^{(n)}} \sqrt{P_2(\underline{y}^{(n)}/0,j)P_2(\underline{y}^{(n)}/1,j)}\, dy_1^{(n)} dy_2^{(n)}$$

Substituting $P_2(\underline{y}^{(n)}/0,j)$ and $P_2(\underline{y}^{(n)}/1,j)$, the last expression reduces to:

$$D = \sum_{j=1}^{N} P(j) \int_0^\infty\!\!\int_0^\infty \rho \frac{2y_0^{(n)} y_1^{(n)}}{\bar{E}_j \bar{E}_{Jj}/\rho} \exp\left\{ - \frac{{y_0^{(n)}}^2}{2}\left| \frac{1}{\bar{E}_j} + \frac{1}{\bar{E}_{Jj}/\rho} \right| \right\} \cdot$$

$$\cdot \exp\left\{ - \frac{{y_1^{(n)}}^2}{2}\left| \frac{1}{\bar{E}_j} + \frac{1}{\bar{E}_{Jj}/\rho} \right| \right\} dy_0^{(n)} dy_1^{(n)}$$

$$= \sum_{j=1}^{N} P(j) \left[\int_0^\infty \sqrt{\frac{2\rho}{\bar{E}_j \bar{E}_{Jj}/\rho}}\; y \exp\left\{ - \frac{y^2}{2} \frac{\bar{E}_j + \bar{E}_{Jj}/\rho}{\bar{E}_j \bar{E}_{Jj}/\rho} \right\} dy \right]^2$$

$$= \sum_{j=1}^{N} P(j) \frac{2\bar{E}_j \bar{E}_{Jj}}{(\bar{E}_j + \bar{E}_{Jj}/\rho)^2} \tag{8.13}$$

Hence:

$$D_{wc} \triangleq \max_{0<\rho\leq 1} D = D(1) = \sum_{j=1}^{N} P(j) \frac{2\bar{E}_j \bar{E}_{Jj}}{(\bar{E}_j + \bar{E}_{Jj})^2}$$

$$= \sum_{j=1}^{N} P(j) \frac{2\bar{E}_{Jj}/\bar{E}_j}{(1 + \bar{E}_{Jj}/\bar{E}_j)^2} \tag{8.14}$$

The worst case jamming is in this case also $\rho=1$. For the uniform channel D_{wc} reduces to:

$$D_{wc} = \frac{2\bar{E}_J/\bar{E}_c}{(1 + \bar{E}_J/\bar{E}_c)^2} \tag{8.15}$$

The last relation is shown graphically in figure 14. It shows the apparently "strange" fact that when the background noise is very low, a ML receiver performs better when the jammer is stronger than the signal,than when the two are equal.In fact, the worst case jamming is obtained when $\bar{E}_J/\bar{E}_c = 1$.

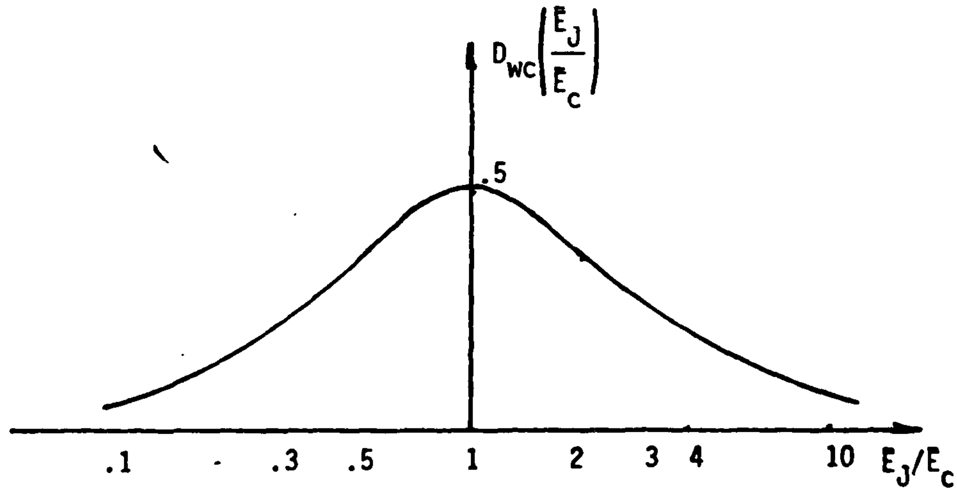


Figure 14. D_{wc} For the Soft Decision Receiver Under Multi-Tone Jamming

8.2.3 Soft Decision receiver having knowledge of ρ over the uniform channel under pulsed / partial band multi-tone jamming.

The input alphabet is: $x \in \{0,1\}$

The output alphabet is: $\underline{y}^{(n)} = (y_0^{(n)}, y_1^{(n)})$; $n=1,\ldots,m$

The conditional probability of $\underline{y}$ given $\underline{x}$ is:

$$P_{2m}(\underline{y}/\underline{x}) = \prod_{n=1}^{m} P_2(\underline{y}^{(n)}/x^{(n)})$$

where:
$$P_2(\underline{y}^{(n)}/x^{(n)}) = (1-\rho)P_2(\underline{y}^{(n)}/x^{(n)}, z_n=0) + \rho P_2(\underline{y}^{(n)}/x^{(n)}, z_n=1)$$

But:
$$P_2(\underline{y}^{(n)}/x^{(n)}, z_n=0) =$$

$$= \frac{2y_{x^{(n)}}^{(n)}}{N_o + \bar{E}_c} \exp\left\{-\frac{y_{x^{(n)}}^{(n)^2}}{N_o + \bar{E}_c}\right\} \frac{2y_{\bar{x}^{(n)}}^{(n)}}{N_o} \exp\left\{-\frac{y_{\bar{x}^{(n)}}^{(n)^2}}{N_o}\right\}$$

and:
$$P_2(\underline{y}^{(n)}/x^{(n)}, z_n=1) =$$

$$= \frac{1}{2} P_2(\underline{y}^{(n)}/x^{(n)}, 1, j_n = x^{(n)}) + \frac{1}{2} P_2(\underline{y}^{(n)}/x^{(n)}, 1, j_n = \bar{x}^{(n)})$$

$$= \frac{1}{2} \frac{2y_{x^{(n)}}^{(n)}}{N_o + \bar{E}_c + \bar{E}_J/\rho} \exp\left\{-\frac{y_{x^{(n)}}^{(n)^2}}{N_o + \bar{E}_c + \bar{E}_J/\rho}\right\} \frac{2y_{\bar{x}^{(n)}}^{(n)}}{N_o} \exp\left\{-\frac{y_{\bar{x}^{(n)}}^{(n)^2}}{N_o}\right\} +$$

$$+ \frac{1}{2} \frac{2y_{x^{(n)}}^{(n)}}{N_o + \bar{E}_c} \exp\left\{-\frac{y_{x^{(n)}}^{(n)^2}}{N_o + \bar{E}_c}\right\} \frac{2y_{\bar{x}^{(n)}}^{(n)}}{N_o + \bar{E}_J/\rho} \exp\left\{-\frac{y_{\bar{x}^{(n)}}^{(n)^2}}{N_o + \bar{E}_J/\rho}\right\}$$

To use the ML metric we take:

$$m(\underline{y}^{(n)};x^{(n)}) = \ell n P_2(\underline{y}^{(n)}/x^{(n)})$$

$$= \ell n \left\{ (1-\rho)P_2(\underline{y}^{(n)}/x^{(n)}, z_n=0) + \right.$$

$$\left. + \frac{\rho}{2}\left[P_2(\underline{y}^{(n)}/x^{(n)}, z_n=1, j_n=x^{(n)}) + P_2(\underline{y}^{(n)}/x^{(n)}, z_n=1, j_n=\bar{x}^{(n)})\right]\right\} \tag{8.16}$$

The Bhattacharyya bound parameter D is then given by:

$$D = \int_{\underline{y}^{(n)}} \sqrt{P_2(\underline{y}^{(n)}/\hat{x}^{(n)})P_2(\underline{y}^{(n)}/x^{(n)})}\, dy_1^{(n)} dy_2^{(n)} \quad ; \hat{x}^{(n)} \neq x^{(n)}$$

$$= \int_{\underline{y}^{(n)}} \sqrt{P_2(\underline{y}^{(n)}/0)P_2(\underline{y}^{(n)}/1)}\, dy_1^{(n)} dy_2^{(n)}$$

Where:

$$P_2(\underline{y}^{(n)}/0) = P_2(y_o^{(n)}, y_1^{(n)}/0)$$

$$= (1-\rho)\frac{2y_o^{(n)}}{N_o+\bar{E}_c}\exp\left\{-\frac{y_o^{(n)2}}{N_o+\bar{E}_c}\right\}\frac{2y_1^{(n)}}{N_o}\exp\left\{-\frac{y_1^{(n)2}}{N_o}\right\} +$$

$$+ \frac{1}{2}\rho\,\frac{2y_o^{(n)}}{N_o+\bar{E}_c+\bar{E}_J/\rho}\exp\left\{-\frac{y_o^{(n)2}}{N_o+\bar{E}_c+\bar{E}_J/\rho}\right\}\frac{2y_1^{(n)}}{N_o}\exp\left\{-\frac{y_1^{(n)}}{N_o}\right\} +$$

$$+ \frac{1}{2}\rho\,\frac{2y_o^{(n)}}{N_o+\bar{E}_c}\exp\left\{-\frac{y_o^{(n)}}{N_o+\bar{E}_c}\right\}\frac{2y_1^{(n)}}{N_o+\bar{E}_J/\rho}\exp\left\{-\frac{y_1^{(n)2}}{N_o+\bar{E}_J/\rho}\right\}$$

and

$$P_2(\underline{y}^{(n)}/1) = P_2(y_o^{(n)}; y_1^{(n)}/1) = P_2(y_1^{(n)}, y_o^{(n)}/0)$$

This is as far as we can go analytically. It is easy to show, however, *that*

$$\lim_{\rho \to o} D = \frac{4(1+\bar{E}_c/N_o)}{(2+\bar{E}_c/N_o\ )^2}$$

which is what we obtain when there is no jamming. Again it seems that $\rho=1$ is the worst case jamming.

Since the ML metric for this receiver requires knowledge of ρ, and is difficult to implement even when ρ is known, we try another approach. It follows from our result on noise jamming, that when no jammer is present, the total ML metric of aSoft Decision receiver is:

$$m(\underline{y};\underline{x}) = \sum_{n=1}^{m} y_{x^{(n)}}^{(n)^2} \tag{8.17}$$

which is jast a sum of squares of the energy detector outputs corresponding to the sequence $\underline{x}$. We want now to find the performance of a receiver using this metric under pulsed / partial band jamming. Using the Chernoff bound we obtain:

$$P(\underline{x}\to\underline{\hat{x}}) \leq E\left[\exp\left\{\sum_{n=1}^{m} \lambda(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2})\right\} \Big/ \underline{x}\right]$$

$$= \prod_{n=1}^{m} E\left[\exp\left\{\lambda(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2})\right\} \Big/ x^{(n)}\right]$$

$$\triangleq \prod_{n=1}^{m} D(x^{(n)},\hat{x}^{(n)};\lambda)$$

where:

$$D(x^{(n)},\hat{x}^{(n)};\lambda) = E\left[\exp\left\{\lambda(y_{\hat{x}^{(n)}}^{(n)^2} - y_{x^{(n)}}^{(n)^2})\right\} \Big/ x^{(n)}\right]$$

$$=E\left[E\left[\exp\left\{\lambda\left(y_{\hat{x}(n)}^{(n)2}-y_{x(n)}^{(n)2}\right)\right\}/x^{(n)},z_n,j_n\right]/x^{(n)}\right]$$

$$=E\left[E\left[\exp\left\{\lambda y_{\hat{x}(n)}^{(n)2}\right\}/x^{(n)},z_n,j_n\right]E\left[\exp\left\{-\lambda y_{x(n)}^{(n)2}\right\}/x^{(n)},z_n,j_n\right]/x^{(n)}\right]$$

$$=\sum_{z,j}P_{z_n,j_n}(z,j)E\left[\exp\left\{\lambda y_{\hat{x}(n)}^{(n)2}\right\}/x^{(n)};z,j\right]E\left[\exp\left\{-\lambda y_{x(n)}^{(n)2}\right\}/x^{(n)};z,j\right]$$

But, for $\hat{x}^{(n)}=x^{(n)}$:

$$E\left[\exp\left\{\lambda y_{x(n)}^{(n)2}\right\}/x^{(n)};z,j\right]=\begin{cases}\dfrac{1}{1-\lambda N_o} & ;\quad z=0\\[2ex] \dfrac{1}{1-\lambda N_o} & ;\quad z=1,\ j=x^{(n)}\\[2ex] \dfrac{1}{1-\lambda(N_o+\bar{E}_J/\rho)} & ;\quad z=1,\ j=\bar{x}^{(n)}\end{cases}$$

$$o<\lambda<\frac{1}{N_o+\bar{E}_J/\rho}$$

and:

$$E\left[\exp\left\{-\lambda y_{x(n)}^{(n)2}\right\}/x^{(n)};z,j\right]=\begin{cases}\dfrac{1}{1+\lambda(N_o+\bar{E}_c)} & ;\quad z=0\\[2ex] \dfrac{1}{1+\lambda(N_o+\bar{E}_c+\bar{E}_J/\rho)} & ;\quad z=1\ ,\ j=x^{(n)}\\[2ex] \dfrac{1}{1+\lambda(N_o+\bar{E}_c)} & ;\quad z=1\ ,\ j=\bar{x}^{(n)}\end{cases}$$

$$o<\lambda$$

Hence, for $\hat{x}^{(n)}\neq x^{(n)}$, $D(x^{(n)};\hat{x}^{(n)};\lambda)=$

$$= \frac{1-\rho}{(1-\lambda N_o)\left[1+\lambda(N_o+\bar{E}_c)\right]} + \frac{\rho/2}{(1-\lambda N_o)\left[1+\lambda(N_o+\bar{E}_c+\bar{E}_J/\rho)\right]} +$$

$$+ \frac{\rho/2}{\left[1-\lambda(N_o+\bar{E}_J/\rho)\right]\left[1+\lambda(N_o+\bar{E}_c)\right]}$$

$$o < \lambda < \frac{1}{N_o+\bar{E}_J/\rho} \tag{8.18}$$

$\triangleq D(\lambda)$

Note that for any allowable value of λ, the first term, namely:

$$\frac{1-\rho}{(1-\lambda N_o)\left[1+\lambda(N_o+\bar{E}_c)\right]}$$

approaches 1 as $\rho \to 0$, and therefore:

$$\min_{o<\lambda<\frac{1}{N_o+\bar{E}_J/\rho}} D(\lambda) \xrightarrow{\rho\to o} 1$$

We conclude that the bound of the Soft Decision receiver using this metric is worthless. We expect that in general the receiver using this metric has poor performance under a low duty cycle jammer. Recall that the same result was derived above for the noise jamming case.

8.3 R_o Evaluation and Simple Applications

Having derived the parameter D for the multi-tone jamming case we can now compute R_o from the equation :

$$R_o = 1-\log_2 1+D_{wc}(\Psi,g) \tag{8.19}$$

8.3.1 BFSK: As for the noise jamming case, the bound on P_b - the bit error probability is:

$$P_b \leq \frac{1}{2} D\left(\frac{\bar{E}_b}{N_o}, \frac{\bar{E}_J}{\bar{E}_b}\right) = $$
$$= \frac{1}{2} D(\Psi_b, g) \qquad (8.20)$$

where: $\Psi_b \triangleq \frac{\bar{E}_b}{N_o}$

For the special case of a Soft Decision receiver over a negligible noise uniform channel, the bound takes the form:

$$P_b \leq \frac{\bar{E}_J/\bar{E}_b}{(1+\bar{E}_J/\bar{E}_b)^2}$$
$$= \frac{g}{(1+g)^2} \qquad (8.21)$$

We want to compare this bound to the exact performance of a ML receiver using the same channel. In this case the operation of the ML receiver can be given a very simple interpretation.

Only two events may occur with nonzero probability :

a. One detector output is zero and the other is nonzero.

b. Both detector outputs are nonzero and one is larger than the other.

When a occurs the ML receiver chooses the nonzero output. When b occurs and $\bar{E}_c > \bar{E}_J$, the receiver chooses the largest output, otherwise the receiver chooses the smaller output. The exact error probability of this receiver is therefore (See Appendix II):

$$P_b = \begin{cases} \frac{1}{2}\frac{\bar{E}_J}{\bar{E}_c+\bar{E}_J} = \frac{1}{2}\frac{g}{1+g} & ; g < 1 \\ \frac{1}{2}\frac{\bar{E}_c}{\bar{E}_c+\bar{E}_J} = \frac{1}{2}\frac{1}{1+g} & ; g \geq 1 \end{cases}$$

The ratio "bound-to-exact" is then:

$$r = \begin{cases} \frac{2g}{1+g} & ; \quad g > 1 \\ \frac{2}{1+g} & ; \quad g \leq 1 \end{cases}$$

which is at most 2.

8.3.2 m Diversity BFSK:

When m diversity is used:

$$P_b \leq \frac{1}{2}\left[D\left(\frac{1}{m}\frac{\bar{E}_b}{N_o};\frac{\bar{E}_J/m}{\bar{E}_b/m}\right)\right]^m \tag{8.22}$$

$$= \frac{1}{2}\,D\left(\frac{\Psi_b}{m};\ g\right)^m \tag{8.23}$$

Using the Hard Decision receiver bound over a uniform channel:

$$D = 2\sqrt{\varepsilon_M(1-\varepsilon_M)}$$

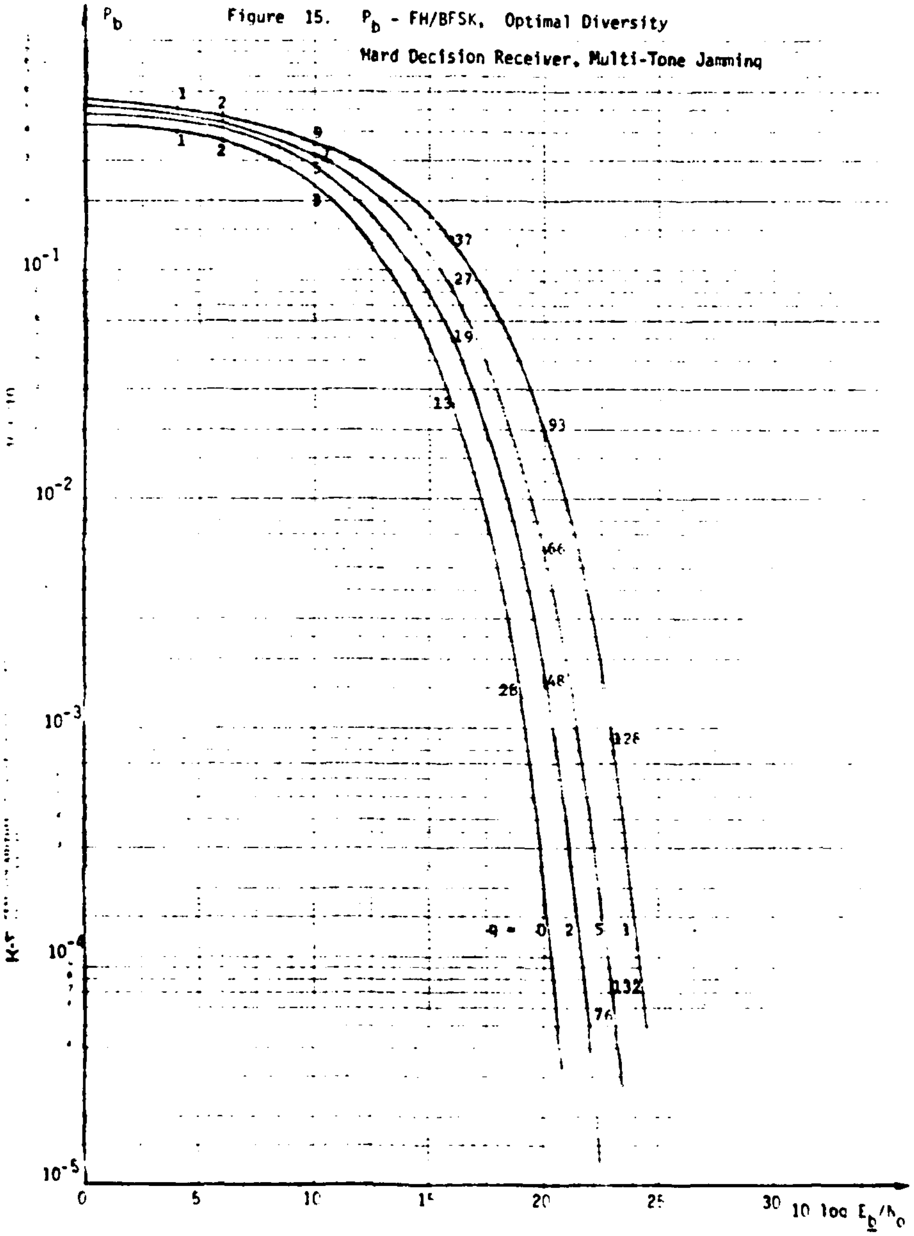
and

$$P_b \leq \frac{1}{2}\left(2\sqrt{\varepsilon_M(1-\varepsilon_M)}\right)^m \tag{8.24}$$

where :

$$\varepsilon_M\left(\frac{\Psi_b}{m};g\right) = \frac{1+\Psi_b g/2m}{2+\bar{E}_b(1+g)/m} \tag{8.25}$$

Figure 15 shows this bound as a function of Ψ for several values of the parameter g,when for each point the optimum value of m was used.


P_b
Figure 15. P_b - FH/BFSK, Optimal Diversity
Hard Decision Receiver, Multi-Tone Jamming
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
5
10
15
20
25
30
10 log E_b/N_0
q = 0 2 5 1
137
127
119
113
93
66
48
28
128
132
76

8.4 Optimum Hopping Strategy over a Nonuniform Channel

We now derive the optimum hopping strategy for an uncoded CS threatened by a multi-tone jammer. As in the noise jamming case, we assume a Slotted Channel as shown in figure 1 . The noise density N_j is equal for all sub-channels: $N_j = N_o$, $j=1,\ldots,L$. The mean received jammer energy per chip of the BFSK signal is defined by the vector

$$\underline{E}_J = (E_{J1}, E_{J2}, \ldots\ldots, E_{JL})$$

where

$$E_{Jj} = \sigma^2_{Jj} T_b \quad , \quad j=1,\ldots,L \; .$$

Here σ^2_{Jj} is the mean received jammer power at the j^{th} sub-channel. The jammer divides his power among the L sub-channels according to:

$$\underline{J} = (J_1, J_2, \ldots\ldots, J_L)$$

Where J_j is the jammer power allocated to the j^{th} sub-channel and:

$$\sum_{j=1}^{L} J_j = J$$

The jammer's propagation loss is defined by:

$$\underline{c} = (c_1, c_2, \ldots\ldots, c_L)$$

where

$$\begin{aligned} E_{Jj} &= \sigma^2_{Jj} T_b \\ &= c_j J_j \end{aligned}$$

The mean received energy of the BFSK signal is defined as before by:

$$\underline{E} = (E_1, E_2, \ldots\ldots, E_L)$$

where we assume that

$$E_1 \geq E_2 \geq E_3 \ldots\ldots \geq E_L .$$

In Appendix II we show that the probability of error of a

BFSK noncoherent receiver over a Rayleigh fading AWGN channel, when a tone jammer is present is:

$$P_b = \begin{cases} \dfrac{N_o+\bar{E}_J}{2N_o+\bar{E}_c+\bar{E}_J} \; ; & \text{when the signal and jammer hit opposite tones} \\[2ex] \dfrac{N_o}{2N_o+\bar{E}_c+\bar{E}_J} \; ; & \text{when the signal and jammer hit the same tone} \end{cases} \tag{8.26}$$

Assuming that the jammer and the signal hit the same tone with probability 1/2 and opposite tones with probability 1/2

$$\bar{P}_b = \frac{1}{2}\,\frac{N_o+\bar{E}_J}{2N_o+\bar{E}_c+\bar{E}_J} + \frac{1}{2}\,\frac{N_o}{2N_o+\bar{E}_c+\bar{E}_J}$$

$$= \frac{N_o+\bar{E}_J/2}{2N_o+\bar{E}_c+\bar{E}_J}$$

$$= \frac{N_o+cJ/2}{2N_o+\bar{E}_c+cJ} \tag{8.27}$$

(In the following we write P_b instead of $\bar{P}_b$)

From this point the derivation follows closely the one given in Chapter 7 for the noise jamming case.

Suppose that the jammer is very weak, so that even if the jammer uses all its power to jam sub-channel 1, it would still have a lower probability of error than sub-channels 2,..,L. Clearly, in this case the CS operator chooses:

$$\underline{P} = (1,0,0,\ldots,0)$$

i.e. no hopping, only sub-channel 1 is used.

Now let J_{t1} be defined by:

$$\frac{N_o+c_1J_{t1}/2}{2N_o+E_1+c_1J_{t1}} = \frac{N_o}{2N_o+E_2}$$

Or:

$$J_{t1} = \frac{2N_o}{c_1}\left(\frac{E_1}{E_2} - 1\right)$$

Then, J_{t1} is the jammer power that when used to jam sub-channel 1, makes it exactly as bad as sub-channel 2 (unjammed). It is intuitively clear, that for $J > J_{t1}$, the CS's operator should hop between sub-channel 1 and other sub-channels. We now give a formal definition of the problem:

Let

$$\underline{P}^* = \left[P^*(1), P^*(2), \ldots\ldots, P^*(L)\right]$$

and

$$\underline{J}^* = \left[J_1^*, J_2^*, \ldots\ldots, J_L^*\right]$$

be the minimax solution, i.e., $\underline{J}^*$ maximizes $P_b(\underline{P}^*, \underline{J})$ over all possible $\underline{J}$ subject to:

$$\sum_{j=1}^{L} J_j = J$$

where

$$P_b(\underline{P}^*, \underline{J}) = \sum_{j=1}^{L} P^*(j)P_{bj}$$

$$= \sum_{j=1}^{L} P^*(j)\,\frac{N_o+c_j/2}{2N_o+E_j+c_jJ_j} \tag{8.28}$$

and $\underline{P}^*$ is the probability vector which minimizes the maximum $P_b(\underline{P}, \underline{J})$ that the jammer can achieve knowing $\underline{P}$. Or:

$$\underline{J}^* = \max^{-1} P_b(\underline{P}^*, \underline{J}) \quad ; \quad \underline{J}: \sum_{j=1}^{L} J_j = J$$

and:

$$\underline{P}^* = \min_{\underline{P}}{}^{-1} \max_{\underline{J}} P_b(\underline{P}, \underline{J})$$

We want to find $\underline{P}^*$ and $\underline{J}^*$.

Using the Lagrange multiplier λ_1 for the constraint:

$$\sum_{j=1}^{L} P(j) = 1 ,$$

Let

$$F_1(\underline{P},\underline{J},\lambda_1) = \sum_{j=1}^{L} P(j) \frac{N_o+c_jJ_j/2}{2N_o+E_j+c_jJ_j} + \lambda_1\left(1-\sum_{j=1}^{L} P(j)\right)$$

Then, the minimax point $(\underline{p}^*,\underline{J}^*)$ mast satisfy the conditions ([3] , Appendix 3B.1 - KUHN TUCKER Conditions) :

$$\left.\begin{aligned} &\left.\frac{\partial F_1(\underline{P},\underline{J}^*,\lambda_1)}{\partial P(j)}\right|_{P(j)=P^*(j)} = 0, \text{ all } j \text{ such that } P^*(j)>o \\ &\sum_{j=1}^{L} P(j) = 1 \end{aligned}\right\} \quad \text{I}$$

Also let:

$$F_2(\underline{P},\underline{J},\lambda_2) = \sum_{j=1}^{L} P(j) \frac{N_o+c_jJ_j/2}{2N_o+E_j+c_jJ_j} + \lambda_2\left(\sum_{j=1}^{L} J_j - J\right)$$

Then, the minimax point should also satisfy the conditions:

$$\left.\begin{aligned} &\left.\frac{\partial F_2(\underline{P}^*,\underline{J},\lambda_2)}{\partial J_j}\right|_{J_j=J_j^*} = 0, \text{ all } j \text{ such that } P^*(j)>o \\ &J_j^* = 0 \qquad , \text{ all } j \text{ such that } P^*(j)=o \\ &\sum_{j=1}^{L} J_j^* = J \end{aligned}\right\} \quad \text{II}$$

From condition I we obtain:

$$\frac{N_o+c_jJ_j^*/2}{2N_o+\bar{E}_j+c_jJ_j^*} - \lambda_1 = 0 \text{ ,all j such that } P^*(j) > 0$$

This implies that at the minimax solution the probability of error P_{bj} of all the active sub-channels must be equal. It also implies that the number of active sub-channels depend on the jammer power J.

It was mentioned above that for $J > J_{t1}$,more than a single sub-channel is used, while for $J \leq J_{t1}$ only sub-channel 1 is used. Similarly we define J_{tn} , $n < L$ to be the highest jammer power for which only n sub-channels are used for hopping. also let J_{tn}^i be the jammer power allocated to sub-channel i when $J = J_{tn}$. Hence:

$$J_{tn} = \sum_{i=1}^{n} J_{tn}^i$$

And by condition I :

$$\frac{N_o+c_1J_{tn}^1/2}{2N_o+\bar{E}_1+c_1J_{tn}^1} = \ldots \quad . = \frac{N_o+c_nJ_{tn}^n/2}{2N_o+\bar{E}_n+c_nJ_{tn}^n} = \frac{N_o}{2N_o+\bar{E}_{n+1}}$$

From which we obtain:

$$J_{tn}^i = \frac{2N_o}{c_i}\left(\frac{\bar{E}_i}{\bar{E}_{n+1}} - 1\right) \quad ; \quad i=1,2,\ldots,n$$

and

$$J_{tn} = \begin{cases} \dfrac{2N_o}{\bar{E}_{n+1}} \displaystyle\sum_{i=1}^{n} \frac{\bar{E}_i}{c_i} - 2N_o \sum_{i=1}^{n} \frac{1}{c_i} & ; \quad n < L \\ \infty & ; \quad n \not< L \end{cases}$$

Therefore, knowing the jammer power, the number N of sub-channels, the CS should use for hopping, is to be found from:

$$J_{t,N-1} < J \leqslant J_{tN} \tag{8.29}$$

We have seen that the minimax solution corresponds to:

$$\frac{N_o + c_j J_j^*/2}{2N_o + \bar{E}_j + c_j J_j^*} = \lambda_1 \quad ; \quad j=1,\ldots,N$$

$$J_j^* = 0 \quad ; \quad j>N$$

Hence:

$$J_j^* = \frac{\lambda_1 \bar{E}_j - N_o(1-2\lambda_1)}{c_j(\frac{1}{2} - \lambda_1)}$$

$$= \frac{2\bar{E}_j \lambda_1}{c_j(1-2\lambda_1)} - \frac{2N_o}{c_j}$$

Summing over all j:

$$J = \frac{2\lambda_1}{1-2\lambda_1} \sum_{j=1}^{N} \frac{\bar{E}_j}{c_j} - 2N_o \sum_{j=1}^{N} \frac{1}{c_j}$$

$$\therefore \qquad \lambda_1 = \frac{1}{2} \frac{J + 2N_o \sum_{j=1}^{N} \frac{1}{c_j}}{\sum_{j=1}^{N} \frac{\bar{E}_j}{c_j} + J + 2N_o \sum_{j=1}^{N} \frac{1}{c_j}}$$

and:

$$J_j^* = \begin{cases} \dfrac{\bar{E}_j}{c_j} \dfrac{J + 2N_o \sum_{j=1}^{N} \frac{1}{c_j}}{\sum_{j=1}^{N} \frac{\bar{E}_j}{c_j}} - \dfrac{2N_o}{c_j} & ; \ j=1,\ldots,N \\ 0 & ; \ j>N \end{cases} \tag{8.30}$$

To find $\underline{P}^*$ we use condition II :

$$\left. \frac{\partial F_2(\underline{P}^*, \underline{J}, \lambda_2)}{\partial J_j} \right|_{J_j = J_j^*} =$$

$$= P^*(j)\ \frac{(2N_o+\bar{E}_j+c_jJ_j)c_j/2 - (N_o+c_jJ_j/2)c_j}{(2N_o+\bar{E}_j+c_jJ_j)^2}\Bigg|_{J_j=J_j^*} + \lambda_2 = 0 \qquad j=1,\ldots,N$$

Or:
$$P^*(j)\ \frac{\bar{E}_jc_j/2}{(2N_o+\bar{E}_j+c_jJ_j^*)^2} + \lambda_2 = 0 \quad ; \quad j=1,\ldots,N$$

Substituting J_j^* from equation 8.30:

$$P^*(j) = -2\lambda_2\ \frac{\bar{E}_j}{c_j}\left(1 + \frac{J + 2N_o \sum_{j=1}^{N} \frac{1}{c_j}}{\sum_{j=1}^{N} \frac{\bar{E}_j}{c_j}}\right)^2$$

Summing over j:

$$\sum_{j=1}^{N} P^*(j) = 1 = -2\lambda_2\left(1 + \frac{J + 2N_o \sum_{j=1}^{N} \frac{1}{c_j}}{\sum_{j=1}^{N} \frac{\bar{E}_j}{c_j}}\right)^2 \sum_{j=1}^{N} \frac{\bar{E}_j}{c_j}$$

$$\therefore \quad \lambda_2 = -\left\{2\left(1 + \frac{J + 2N_o \sum_{j=1}^{N} \frac{1}{c_j}}{\sum_{j=1}^{N} \frac{\bar{E}_j}{c_j}}\right)^2 \sum_{j=1}^{N} \frac{\bar{E}_j}{c_j}\right\}^{-1}$$

Hence:

$$P^*(j) = \begin{cases} \dfrac{\frac{\bar{E}_j}{c_j}}{\sum_{i=1}^{N} \frac{\bar{E}_i}{c_i}} & ; \quad j=1,\ldots,N \\ 0 & ; \quad j > N \end{cases} \qquad (8.31)$$

Equation 8.31 and its parallel for the noise jamming case, equation 7.13 have exactly the same form. Note however, that the set of constants c_j, $j=1,\ldots,L$ was differently defined, reflecting the fact that in the multi-tone case the jammer too is subject to fading. In contrast, equation 8.30 and 7.12 are not identical and therefore, result in a different jammer power distribution.

We must still show that this procedure leads to the required minimax point, i.e., that:

$$\underline{J}^* = \max_{\underline{J}}{}^{-1} P_b(\underline{P}^*,\underline{J}^*)$$

and that

$$\underline{P}^* = \min_{\underline{P}}{}^{-1} \max_{\underline{J}} P_b(\underline{P},\underline{J})$$

Let us show first that the extremum point of $P_b(\underline{P}^*,\underline{J})$ at $\underline{J} = \underline{J}^*$ is a maximum and that $P_b(\underline{P}^*,\underline{J})$ exceeds any other value of $P_b(\underline{P}^*,\underline{J})$.

$$\frac{\partial P_b(\underline{P}^*,\underline{J})}{\partial J_j} = P^*(j)\frac{\bar{E}_j c_j/2}{(2N_o+\bar{E}_j+c_jJ_j)^2}$$

$$\therefore \quad \frac{\partial^2 P_b(\underline{P}^*,\underline{J})}{\partial J_j^2} = P^*(j)\,\frac{-(\bar{E}_j c_j)}{(2N_o+\bar{E}_j+c_jJ_j)^3} < 0 \quad ; \quad \text{for all } 0<J_j$$

To show that

$$\underline{P}^* = \min_{\underline{P}} \max_{\underline{J}} P_b(\underline{P},\underline{J})$$

recall that when $\underline{J} = \underline{J}^*$, all the active sub-channels have the same error probability whereas the non active sub-channels either exceed or equal that level. Hence:

$$P_b(\underline{P}^*,\underline{J}^*) \leq P_b(\underline{P},\underline{J}^*) \quad \text{for every probability vector } \underline{P}.$$

Clearly:

$$P_b(\underline{P},\underline{J}^*) \leq \max_{\underline{J}} P_b(\underline{P},\underline{J})$$

$$\therefore \quad P_b(\underline{P}^*,\underline{J}^*) \leq \max_{\underline{J}} P_b(\underline{P},\underline{J})$$

But, by definition:

$$P_b(\underline{P}^*,\underline{J}^*) = \max_{\underline{J}} P_b(\underline{P}^*,\underline{J})$$

Hence

$$\max_{\underline{J}} P_b(\underline{P}^*,\underline{J}) \leq \max_{\underline{J}} P_b(\underline{P},\underline{J}) \qquad \text{for every}$$

probability vector $\underline{P}$.

$$\therefore \quad \underline{P}^* = \min_{\underline{P}}{}^{-1} \max_{\underline{J}} P_b(\underline{P},\underline{J})$$

The last result can be applied equally well to the coded systems discussed above. The proof is very much the same as that given for noise jamming, and was therefore ommited.

CHAPTER IX

CONCLUDING REMARKS

The performance of FH/MFSK noncoherent CSs over an HF Rayleigh fading channel, which is subject to jamming, has been studied. A general error bound was used to evaluate and compare the performance of several coded CSs under noise and multi-tone jamming. The same bound was used also to optimize certain receiver parameters. The receivers studied are basically conventional noncoherent FH/MFSK receivers, which for every chip time-interval generate M matched filter output signals. The receivers differ, though, in the type of processing these M signals undergo and the metric used by the decoder.

For each receiver the "worst case jamming" was found in terms of ρ_{wc}, which is either the jammed fraction of the total spread spectrum bandwidth, in case of partial-band jamming, or the duty cycle of a pulsed jammer. The cutoff rate R_0 was then derived for each receiver under its worst case jamming condition. ρ_{wc} depends, in general, on the receiver being used. For a Hard Decision receiver, ρ_{wc} is one, whether JSI is available or not. Soft Decision receivers may exhibit worst performance at low values of ρ. It has been shown that the simple squared matched filter output metric, which is optimal for broadband / continuous jamming (and, of course, for non-jammed uniform channels) results in a poor performance when ρ is small. Under this kind of jamming, the Hard Decision receiver is a better choice. A similar situation may arise when a Soft Decision receiver, having no CSI, hops over a nonuniform

channel. A Soft Decision receiver using the squared matched filter output metric performs better than the Hard Decision receiver, provided that, JSI is available, or when ρ can be measured and the ML metric implemented. Both techniques may add considerably to the overall complexity of the CS. Another approach has been demonstrated by introducing the Quantizer-Limiter and the Limiter receivers. These easy to implement receivers seem to outperform the Hard Decision receiver, provided that a certain receiver parameter is tuned to its optimal value, which depends on the noise spectral density and the mean signal power. It has been shown, though, that the performance is relatively insensitive to changes in the value of this parameter.

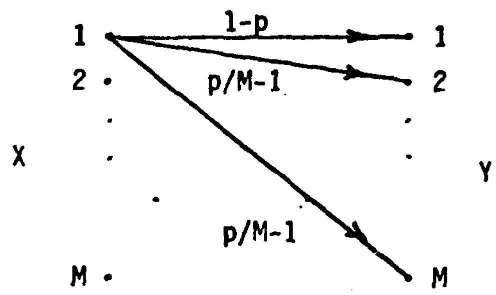
As well known [4,5] diversity, which in effect transmits each coded bit over many frequency sub-bands, which fade independently, may be extremely beneficial over fading channels. This is also the case when combatting jammer is the issue. The full impact of optimal diversity for the Soft Decision FH/MFSK receiver under noise jamming can be seen when comparing figures 12 and 13a. Typically though, in practical situations there is a constraint imposed on the minimal chip duration, limiting the maximal order of diversity. In extreme cases the minimal T_c is equall to T_b, i.e., for $M=2$ no diversity (or coding) is possible. In such a situation high M systems have an impressive advantage over binary systems. On the other hand, binary systems may perform better than high M systems under multi-tone jamming. Such jammers, however, require more jamming equipment and more information about the target CS. A system in which M can be field selected according to the kind of threat and variable field con-

ditions seems very attractive.

APPENDIX I

Derivation of the Symbol Error Probability of a Noncoherent Orhogonal MFSK System Over a Rayleigh Fading AWGN Channel.

The situation is represented schematically by the following model



The input and output alphabets are $X = Y \in \{1,\ldots,M\}$. Assuming that $x=1$ was sent, the probability of symbol error is :

$$P = P_r\{y_1 \leqslant y_i \text{ for some } i \neq 1/x=1\}$$

Where y_k is the output of the receiver's k^{th} energy detector. Hence:

$$P = 1 - P_r\{y_i \leqslant y_1 \text{ for all } i \neq 1/x=1\}$$

$$= 1 - \int_0^\infty P(y_i < \alpha \text{ for all } i \neq 1/x=1) P_{y_1}(\alpha/x=1) d\alpha$$

$$= 1 - \int_0^\infty [1-P(y_i \geqslant \alpha/x=1)]^{M-1} P_{y_1}(\alpha/x=1) d\alpha$$

But, y_i is a Rayleigh random variable having the probability distri-

bution [4]:

$$P_{y_i}(y/x=1) = \frac{2y}{N_o} \exp\left\{-\frac{y^2}{N_o}\right\} \quad ; \ i\neq 1, \ y>0$$

Where N_o is the one-sided spectral density of the received noise.

Also

$$P_{y_1}(y/x=1) = \frac{2y}{N_o+\bar{E}_c} \exp\left\{-\frac{y^2}{N_o+\bar{E}_c}\right\} \qquad y>0$$

Where $\bar{E}_c$ is the mean received energy per symbol.

Hence, for $i\neq 1$:

$$P_r\{y_i > \alpha/x=1\} = \exp\left\{-\frac{\alpha^2}{N_o}\right\}$$

$$\therefore \qquad [1 - P(y_i > \alpha/x=1)]^{M-1} = \left[1-\exp\left\{-\frac{\alpha^2}{N_o}\right\}\right]^{M-1}$$

$$= \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \exp\left\{-\frac{k\alpha^2}{N_o}\right\}$$

Therefore:

$$P = \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \int_0^\infty \exp\left\{-\frac{k\alpha^2}{N_o}\right\} \frac{2\alpha}{N_o+\bar{E}_c} \exp\left\{-\frac{\alpha^2}{N_o+\bar{E}_c}\right\} d\alpha$$

$$= \sum_{k=1}^{M-1} \binom{M-1}{k} (-1)^{k+1} \frac{1}{1+k(1+\bar{E}_c/N_o)}$$

For M=2 this reduces to:

$$P = \frac{1}{2 + \bar{E}_c/N_o}$$

APPENDIX II

Derivation of the Symbol Error Probability of a Noncoherent Orthogonal BFSK System over a Rayleigh Fading AWGN Channel Hit by a Multi-Tone Jammer.

We assume that during each chip time-interval the jammer hits either ω_o or ω_1 , when ω_o and ω_1 are the two designated tone positions of the BFSK receiver. Let the random variable j be defined as:

$$j = \begin{cases} 1 & ; \quad \text{the jammer hits } \omega_1 \\ 0 & ; \quad \text{the jammer hits } \omega_o \end{cases}$$

Then, assuming hypothesis H_o is true, r_{co} and r_{so} (see figure 2) are independent zero mean Gaussian random variables having the common varians :

$$E\{r_{co}^2\} = E\{r_{so}^2\} = \begin{cases} N_o + E_c & ; \ j=1 \\ N_o + E_c + E_J & ; \ j=o \end{cases}$$

Likewise, r_{c1} and r_{s1} are independent zero mean Gaussian random variables having the common varians:

$$E\{r_{c1}^2\} = E\{r_{s1}^2\} = \begin{cases} N_o + E_J & ; \ j=1 \\ N_o & ; \ j=0 \end{cases}$$

Defining:

$$Z_o \triangleq \sqrt{r_{co}^2 + r_{so}^2}$$

$$Z_1 \triangleq \sqrt{r_{c1}^2 + r_{s1}^2}$$

We have:

$$P_{Z_o/j}(z/1) = \frac{2z}{N_o+\bar{E}_c} \exp\left\{-\frac{z^2}{N_o+\bar{E}_c}\right\}$$

$$P_{Z_o/j}(z/0) = \frac{2z}{N_o+\bar{E}_c+\bar{E}_J} \exp\left\{-\frac{z^2}{N_o+\bar{E}_o+\bar{E}_J}\right\}$$

$$P_{Z_1/j}(z/1) = \frac{2z}{N_o+\bar{E}_J} \exp\left\{-\frac{z^2}{N_o+\bar{E}_J}\right\}$$

$$P_{Z_1/j}(z/0) = \frac{2z}{N_o} \exp\left\{-\frac{z^2}{N_o}\right\}$$

Hence:

$$\begin{aligned} P(\text{error}/H_o, j=1) &= P(Z_1 \geq Z_o/H_o, j=1) \\ &= \int_0^\infty P(Z_1 \geq Z_o/H_o, j=1, Z_o=z) P_{Z_o}(z/H_o, j=1)dz \\ &= \int_0^\infty P(Z_1 \geq z/H_o, j=1) P_{Z_o}(z)dz \end{aligned}$$

But:

$$\begin{aligned} P(Z_1 > z) &= \int_z^\infty P_{Z_1}(z/H_o, j=1)dz \\ &= \int_z^\infty \frac{2z}{N_o+\bar{E}_J} \exp\left\{-\frac{z^2}{N_o+\bar{E}_J}\right\} dz \end{aligned}$$

$$= \exp\left\{-\frac{z^2}{N_o+\bar{E}_J}\right\}$$

$$\therefore \quad P(\text{error}/H_o, j=1) = \int_o^{\infty} \exp\left\{-\frac{z^2}{N_o+\bar{E}_J}\right\} \frac{2z}{N_o+\bar{E}_J} \exp\left\{-\frac{z^2}{N_o+\bar{E}_c}\right\} dz$$

$$= \frac{N_o + \bar{E}_J}{2N_o+\bar{E}_c+\bar{E}_J}$$

Similarly,

$$P(\text{error}/H_o, j=0) = P(Z_1 \geqslant Z_o/H_o, j=0)$$

$$= \int_o^{\infty} P(Z_1 \geqslant Z_o/H_o, j=0, Z_o=z) P_{Z_o}(z/H_o, j=1) dz$$

$$= \frac{N_o}{2N_o+\bar{E}_c+\bar{E}_J}$$

Hence:

$$P(\text{error}/H_o) = P(\text{error}/H_o, j=1)\, P_j(1) + P(\text{error}/H_o, j=0) P_j(0)$$

$$= P_j(1)\, \frac{N_o + \bar{E}_J}{2N_o+\bar{E}_c+\bar{E}_J} + P_j(0) \frac{N_o}{2N_o+\bar{E}_c+\bar{E}_J}$$

By symmetry:

$$P(\text{error } H_1) = P_j(0)\, \frac{N_o + \bar{E}_J}{2N_o+\bar{E}_c+\bar{E}_J} + P_j(1)\, \frac{N_o}{2N_o+\bar{E}_c+\bar{E}_J}$$

Assuming now that π_o and π_1 are the apriori probabilities of H_o and H_1, we obtain:

$$P_s = P(\text{error}/H_1)\Pi_1 + P(\text{error}/H_o)\Pi_o$$

$$= \Pi_1 \frac{N_o + \bar{E}_J}{2N_o+\bar{E}_c+\bar{E}_J} + \Pi_o \frac{N_o}{2N_o+\bar{E}_c+\bar{E}_J}$$

Assuming also that $\Pi_1 = \Pi_2 = ½$, we obtain:

$$P_s = \frac{N_o + \bar{E}_J/2}{2N_o+\bar{E}_c+\bar{E}_J}$$

BIBLIOGRAPHY

1. Viterbi, A.J., and Jacobs, I.M., "Advances in Coding and Modulation for Noncoherent Channels Affected by Fading, Partial Band, and Multiple Access Interference", in Advances in Communication Systems, Vol. 4. New York: Academic Press, 1975, op. 279-308.

2. Omura, J.K., "Principles of Secure Digital Communication", A National Short Course, Hayatt Hotel, Los Angeles International Airport, June 23-27, 1980.

3. Viterbi, A.J., and Omura, J.K., Principles of Digital Communication and Coding, New York: McGraw-Hill, 1979.

4. Wozencraft, J.M. and Jacobs, I.M., Principles of Communication Engineering. New York: Wiley, 1965, pp. 533-540.

5. Schwartz, M., Bennett, W.R., and Stein, S., Communication Systems and Techniques, McGraw-Hill Book Co., 1966.

6. Whalen, A.D., Detection of Signals in Noise, New York: Academic Press, 1971.

7. Martin, D.R., Convolutional Code Performance with Optimal Jamming, TRW Interoffice Correspc nce, 80-7132-70611-001, February 1980.

8. Viterbi, A.J., "Spread Spectrum Communications - Myths and Realities", IEEE Commun. Soc. Mag., vol. 17, pp. 11-18, May 1979.

9. Goldberg, B., "300 kHz-30 MHz MF/HF", IEEE Trans. COmmun. Technol., vol. COM-14, pp. 767-784, Dec. 1966.

10. Special Issue on Spread Spectrum Communications, IEEE Trans. on Commun. Technol. vol. COM-25, no. 8, Aug., 1977.

11. Dixon, R.C., Spread Spectrum Systems, New York: Wiley, 1976.

12. Ultant, W.F., "Spread Spectrum: Principles and Possible Application to Spectrum Utilization and Allocation", IEEE Commun., Soc. Mag., vol. 16, pp. 21-31, Sept. 1978.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